Synthesis of Transients in Guitar Sounds

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Abstract
Guitar sounds are analyzed in detail to understand the origin of transients produced during the attack and damping of a note. These transients and the main note are produced on the computer using physical modeling and FM synthesis. The implementation successfully simulates the articulated multiple note sounds for a complete set of performance parameters which correspond to various playing techniques. With appropriate adjustment in the parameters, the model can easily be used for the synthesis of all plucked instrument sounds.

1 Introduction
The guitar, a very popular musical instrument, is also a very interesting physical system for studying acoustics, due to the coexistence of the vibrational modes supported by its three dimensional body, its extremely non-ideal strings, and a complicated excitation mechanism. The picture becomes even more complicated when the coupling between these elements is incorporated into the mathematical model which is built to represent the instrument and synthesize its sound.

Within the last few years, detailed research has been done on the topic. Karplus and Strong proposed an intuitive digital synthesis algorithm [1] which is very useful in making the connection between physical models and digital signal processing, in particular waveguide synthesis. Karjalainen and Smith proposed various techniques for the modeling of the body, when it is decoupled from strings [2]. Chaigne used finite difference methods to study the two transverse vibrational modes of a string, their coupling to the bridge, and their mutual coupling [3].

All of these models are very successful in analyzing one or few aspects of the instrumental sound and synthesizing it. It is a well known fact, however, that the perceived individuality of a musical instrument is only partly related to the main note and its harmonics. The sounds whose durations are much shorter than that of the note itself - referred to as "transients" in this paper - are crucial in a synthesis model which is designed to produce sounds to be perceived as "real". Surprisingly, they have usually been neglected in sound synthesis especially in commercial synthesizers.

In this paper, the properties of these transients are explained and it is shown that they are not simply "noise" but have a harmonic or inharmonic spectrum. Then a hybrid model which applies previously successful methods to their synthesis and the synthesis of articulated note sounds is presented.

2 Qualitative Analysis of Transients and Articulation
Fig. 1 shows the waveform for three open string G’s played successively on a classical guitar. The portion \( t < t_0 \) in the figure represents the end portion of the first note. After \( t_0 \), the plucking device - the nail in this context - slides on the string, releasing it at time \( t = t_{ba} \). The duration of this sliding motion depends on the angle of the finger from the string and its speed which is the variable determining the amplitude of the actual note. In this time interval \( (t_0 < t < t_{ba}) \), a very soft "scratch" sound is heard.

The analysis of the spectrum of this sound shows that it is harmonic with a wideband noise superimposed (Fig. 2 (c)). This is the longitudinal mode vibration of the string the details of which will be given in the next Sec. 3.1. At \( t = t_{ba} \) (body attack) the transverse vibration starts along with a percussive body sound "tap sound". In classical guitars, the amplitude of the tap sound is a function of the angle between the finger and the top plate and the overall amplitude of the note. For so called "free stroke" in which the finger moves more or less parallel to the top plate,
Figure 1: The sound samples from three successive G notes played on a classical guitar. Only the second note is completely included in the figure.

its amplitude is smaller. At \( t = t_s \), the tap portion dies away, and only the two transverse modes persist. The interval \( t_s - t_{ba} \) is usually less than one second. It should be noted that if a tap sound is produced by damping the transverse modes, its duration is about 0.5 seconds. This means that when the transverse modes are excited, they transfer some of their energy into the body, which produces a convolution effect spreading the time duration of the tap.

Between \( t_d \) and \( t_{bd} \) (body damp) the main note is damped and a very short buzzing sound is heard. This time interval is generally less than 10 milliseconds. At \( t_{bd} \) two crucial transients begin to be generated. One of them is a second tap sound resulting from the motion of the finger towards the top plate during damping. The amplitude of this tap is highly dependent on the speed and position of the finger. As it moves closer to the bridge, the amplitude gets close to that of the tap excited at \( t = t_{ba} \). As the finger moves away from bridge or as it moves more parallel to the top plate to play the next note (free stroke), the amplitude decreases. In steel string guitars, the position of the plucking device -plectrum in this context- does not matter as much because of the higher tension on the string.

The second crucial transient is the transverse vibration of the divided string. The position of the plucking device determines the two pitches heard at this time. The material properties of the string, on the other hand, determine their amplitudes. In particular, for a plectrum, both portions of the string can be heard simultaneously. In classical guitars, if the thumb provides the excitation, the portion on the bridge side is heard since the skin of the finger damps the nut portion. If any other finger excites the string, the nut portion is heard since the bridge portion is damped by the skin. This explains why notes played with the thumb are perceived differently than those played with other fingers in classical guitars.

The coexistence of these two transients can be investigated by comparing the spectra at time instants \( t_3 \) (Fig. 2 (a)) and \( t_4 \) (Fig. 2 (b)). As soon as the vibrating G is damped at \( t = t_{bd} \), the fundamental moves from about 193 Hz to 205 Hz, and only the nut portion of the string continues to vibrate since the index finger excites the string. The ratio \( 1/20 \) of these two frequencies gives the percent distance of the finger from the bridge. At \( t_4 \), the observed Helmholtz mode at 105 Hz demonstrates the percussive body tap sound generated by the damping.

3 Synthesis of Transients

3.1 Longitudinal Mode of the String

The fundamental frequency of the longitudinal mode of vibration is given by\(^1\):

\[
f_0 = \frac{1}{2L} \sqrt{\frac{E}{\rho_0}}.
\]

Here \( f_0 \), \( E \), \( \rho_0 \) and \( L \) are the fundamental frequency, Young's Modulus, mass density and string length respectively. The frequency is tension independent to first order. This is consistent with the experimental observation that, even if the tuning of a string is changed, the fundamental frequency in \( t_0 < t < t_{ba} \) remains constant. As pointed out by Chaigne, the Young's modulus itself can vary over time and depends on the tension the string is set at \([4]\). Using \( E = 4.5 \times 10^9 N/m^2 \) and \( \rho_0 = 1.067 g/cm^3 \) for the G string vibration in a classical guitar, Eqn. 1 gives \( f_0 = 1600 \text{ Hz} \). The experimental value obtained from the samples shown in Fig. 1 is 1546 Hz (Fig. 2 (c)). These and other numerical values establish the fact that the sliding motion of the plucking device indeed excites the longitudinal mode.

\(^1\)The details of this calculation will be published elsewhere.
The synthesis model is very similar to that proposed by Cook [5] for flutes. An enveloped white noise is supplied to a loop containing a delay line and a one-pole filter. From recordings, the envelope is found to be exponential, (or linear in dB scale) with a typical duration between 20 and 50ms. The length of the delay line is tuned to obtain the pitches extracted from recordings. The noise represents the roughness of the string and its amplitude is made adjustable since as the string gets to be played over time, the roughness increases.

### 3.2 Transverse Modes of the String

Finite difference methods are very powerful in synthesizing the transverse vibrational modes. In this implementation, the special case $\Delta x = c \Delta t$ is used to discretize the ideal string equation. This choice introduces numerical dispersion for non-ideal strings [3] but it is satisfactory for demonstrating the ideas about articulations. This way, waveguides can be used to represent the string portions and the most significant loss terms, $-2b_1 \frac{\partial^2 y}{\partial x^2} + 2b_2 \frac{\partial^3 y}{\partial x^3}$ [3] can be modeled by a biquad filter assuming $\alpha L \ll 1$, where $\alpha$ is the loss per unit length of the string and $L$ is the string length.

Fig. 3 shows an implementation for the transverse modes. The four delay lines model the propagation of the rightgoing, $y^+(x,t)$ and leftgoing $y^-(x,t)$ components of the string displacement $y(x,t)$. The scattering junction in the middle represents the plucking device. The "nut filter" consists of several cascaded filters: a biquad filter to simulate the losses, and two first order allpass filters to tune the pitch and generate inharmonicity by introducing phase delays [6].

At time $t_{ba}$, the waveguides are filled with sloped lines which simulate the initial shape of the string. As the output signal, the sum of values on both sides of the nut filter is used. Although this is an intuitive choice, the spectrum exhibits missing harmonics at the correct frequencies, as required. The scattering function parameters are adjusted according to the time intervals and the corresponding transients shown in Fig. 1. For $t_{ba} < t < t_d$, the transmission coefficients in both directions are $1.0$ and the reflection coefficients are $0.0$. As soon as the string starts to be damped, the plucking device constraints the displacement. This is implemented by comparing the sum of the displacement at the junction and the constraint. If the displacement is larger, it is clipped and the leftgoing and rightgoing components are scaled to preserve their mutual ratios. The constraint is assumed to be slowly varying in time, in other words, its characteristic time scale is larger than the maximum period in the string vibration. The string displacement becomes zero at $t = t_{ba}$. It then quickly becomes negative and excites the divided portions of the string. Right around this moment, the reflection coefficients are increased and the transmission coefficients are decreased to a value close to zero such as $0.05$. Depending on the excitation, they can be made non-symmetric, for example to introduce additional losses, the sum of the reflection and transmission coefficients on one side can be made less than $1.0$. If the nut side is chosen, the model simulates a thumb finger, which damps the nut portion of the string.

The two transverse modes are coupled to each other at the bridge using another scattering junction. The parameters in this junction are not modified during the course of a note. Moreover, in the current implementation, the parameters in this junction are kept constant in frequency. For better results, a frequency dependent admittance matrix should be used.

### 3.3 Body Vibration

The body vibration, $b(t)$, is assumed to be known—indeed estimated—and can be generated by FM synthesis or any other method. It is coupled to the transverse modes on the bridge, as a boundary condition: $b(t) = y^+(x = \text{bridge},t) + y^-(x = \text{bridge},t)$. This approach works very well if a tap sound is recorded without the transverse modes, and then synthesized with an envelope more spread in time to account for the continuous excitation due to the vibration string. In this paper, two sets of FM generators were used for
the tap sounds generated at $t_{bd}$ and the corresponding $t_{ba}$ of the third note. Their carrier envelopes are added to simulate the frequent excitation of the body in fast passages.

## 4 Synthesis of Articulations

The output sound is obtained by enveloping, timing and smoothly adding (fade in/out) all the vibrational mode sounds described in Sec. 3. With the use of a more complicated timing circuit, the ideas represented here can be used for the digital synthesis of sounds produced using various playing techniques specific to any plucked string instrument. For example, in the case of a classical guitar, harmonics can be synthesized by adding another scattering junction and scaling the amplitudes of the signals. Ascending slurs which also produce a percussive body sound can simply be implemented by moving the scattering junction to the location where the finger "hammers on" and applying a constraint for the displacement. All the left hand "squeak" sounds which are a combination of longitudinal modes and noise due to string roughness, can be generated by adjusting the timing between the individual elements of the model.

## 5 Conclusions

The sound data produced using the procedure described in this paper is perceived to be very close to the reality especially when it consists of articulated guitar notes. It is very important to notice that this result is achieved not by synthesizing the transverse or body modes very accurately, but by incorporating the correct transients into the synthesis model.

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## References


