Synthesis of the trumpet tone based on physical models

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Abstract

This paper deals with sound production in brass instruments. Our main interest here is the excitation system, i.e. the interaction between lips and instrument. The two one-dimensional models traditionally employed to describe the physics of the lips, known as outward-striking and upward-striking, have been showed [8] to be unsuitable to fully explain the behavior of the lips. Starting from anatomical considerations, we developed a two-dimensional model containing both the one-dimensional configurations and whose properties are accurate enough to represent the observed duality.

1 Introduction

Traditional physical modeling of lips as reed in brass instruments refers to two models: the first, called swinging door or outward striking, assumes that lips close when the mouthpiece (acoustic) pressure increases; the second, referred to transverse or upward striking model, represents lips as a valve which opens either with a mouth pressure increase or an acoustical pressure increase. Recent measurements [8] showed that instruments like trumpet and trombone support both the outward striking and an upward striking modes so that they are both needed to fully model the acoustical behavior of the lips.

Starting from an anatomical approach, which considers the muscular configuration of the face, we developed a model of lips with two degrees of freedom which contains, as particular cases, both the outward and the upward striking models.

2 Two one-dimensional models

Detailed description and simulations for each one-dimensional valve configuration can be found for example in [1]. Here we just briefly summarize the most important differences between them in order to make the nature of the problem clearer. A mathematical analysis based on linearized models leads to Table 1, which compares phase and frequency relationships in the two cases: \( f, f_{\text{lip}}, f_p \) are respectively sound frequency, lip resonance frequency and input impedance peak frequency, \( Y_{\text{lip}}(f) \) is the equivalent lip admittance, \( Z_{\text{in}}(f) \) is the instrument input impedance and \( D(f)=X(f)/P(f) \), in which \( X(f) \) is lip opening and \( P(f) \) is mouthpiece pressure, is the dynamic compliance [4].

Yoshikawa [8] measured \( \angle D(f) < 0 \) for the second resonance mode of the air column and \( \angle D(f) > 0 \) for the higher modes, so that the outward striking configuration is supported for the second mode while the upward striking is dominant for the others.

<table>
<thead>
<tr>
<th>Upward striking</th>
<th>Outward striking</th>
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<tbody>
<tr>
<td>( f &lt; f_{\text{lip}} )</td>
<td>( f &gt; f_{\text{lip}} )</td>
</tr>
<tr>
<td>( f &lt; f_p )</td>
<td>( f &gt; f_p )</td>
</tr>
<tr>
<td>( \angle D(f) \equiv -\pi / 2 )</td>
<td>( \angle D(f) \equiv \pi / 2 )</td>
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</table>

Table 1. Comparison between outward striking and upward striking frequency and phase relationships.

A consequence of this duality is that the sound frequency for a real instrument is less than the impedance peak frequency for lower modes and higher for the others. Therefore, a one-dimensional model cannot sound in tune unless we alter ad hoc the frequency response of the resonator.

3 A two-dimensional model

3.1 An anatomic approach

Mechanical parameters are strongly related with muscular actions; if we consider the structure of labial musculature (figure 1) we notice that several muscles can affect the properties of the lip, nevertheless the main actions are due to three classes of muscles:
- Buccinator: this muscle pull the lips laterally.  
- Orbicularis: it’s arranged around the lips and is responsible for the concentric contraction of the lips.  
- Other cheek and jaw muscles: these muscles radiating out of the mouth have the opposite action of the orbicularis.
In figure 2a we consider a simplified transversal section of the upper lip, where the gray arrows show the muscular tensions.

Figure 1: Labial musculature.

To relate these muscular controls with lip motion we will represent the lip as a semi-rigid bar, that is compressible only on its length, which can rotate around the point where the lip leans on the entrance ring of the mouthpiece. A more accurate description should consider distributed parameters; here we will accept a compromise, modeling the longitudinal stiffness as a single spring and thinking of a uniformly distributed mass. The assumption of a semi-rigid bar is reasonable because lips move in a lump [5].

A mechanical scheme for this model, together with its parameters, is shown in figure 2b.

Figure 2: a) Simplified section of the upper lip, b) two-dimensional mechanical model.

The buccinator pulls the lips laterally; like in a string, strain is related to the natural frequency of the system \( f_{lip} \); in our model, a strain increase corresponds to an increase of the rotational stiffness \( k_\theta \). Moreover, when the buccinator is stretched, part of the lip is extruded from the mouthpiece so that the effective lip mass diminishes; this also contributes to increase \( f_{lip} \). The antagonistic action of the orbicularis and the other muscles determines the longitudinal parameters: again, for high strain we have a stiffness increase.

### 3.2 Fluidynamical equations

The main non linearity of the model is the relationship between flow and pressure. Following Adachi and Sato’s approach [1] we get the following equations:

\[
p_0 - p = \frac{\rho u^2}{2 A(x)}
\]

in which \( \rho \) is air density and \( A(x) \) is the effective area given by

\[
A(x) = \left[ \frac{1}{(bx)^2} \frac{2}{S_{me} bx} + \frac{2}{S_{me}^2} \right]^{\frac{1}{2}}
\]

where \( S_{me} \) is the mouthpiece entrance area.

### 3.3 Motion equations

We can describe the motion of the lip through two non linear differential equations whose variables are length \( l \) and inclination \( \theta \) of the lip. The excitation pressures are controlled by the fluidynamical equation seen in the previous section.

Since mass is uniformly distributed over the lip, compression acts on an effective barycentric mass whose value is set to half of the actual value (or equivalently the associated displacement is \( l/2 \)).

Longitudinal equation is therefore:

\[
m \frac{d^2 l}{dt^2} + r_l l + k_l (l - l_0) = \left[ - p_B \cos(\theta) - p \sin(\theta) \right] \dot{\theta} + \frac{m \theta^2}{2} l + l(x) k_s \dot{\theta} \cos(\theta)
\]

where, in the left hand side \( r_l \) is the longitudinal damping factor, \( k_l \) is the longitudinal stiffness and \( l_0 \) is the equilibrium length. The first term on the right hand side represents the external forces due to Bernoulli pressure \( p_B = \rho u^2/(2bs) \) and \( p \); the second term is the centrifugal force and the third, where \( k_s \) is an additional stiffness, was introduced to manage lips collision to avoid discontinuities in the force term.

The rotational equation was deduced applying the angular momentum variation law:

\[
\dot{M} = \sum \tau_i
\]

lip angular momentum is

\[
M = \frac{\dot{\theta} m l^2}{3}
\]

which, differentiated and substituted into (4) yields:
\[
\frac{1}{3} m \ddot{\theta} + \frac{2}{3} l l m = (p_0 - p) \frac{l^2 b}{2} + l(x)p_h \sin(\theta) sbl - r_0 \theta - k_0(\theta - \theta_0) - l(-x)k_x x \sin(\theta) l
\]

with the assumption that \( p_0 \) and \( p \) act on an area \( b l \), while Bernoulli pressure \( p_h \) acts on \( b s \). The meaning of the terms on the right hand side is analogous to that discussed for the longitudinal equation; here \( k_0 \) is rotational stiffness and \( r_0 \) is rotational damping factor.

Lip opening \( x \) depends on \( l \) and \( \theta \) through the formula:

\[
x = 2[l_x - l \cos(\theta)]
\]

which is the link between motion and fluidynamical equations.

4 Implementation of the model

The implementation of the resonator was based on the approximated dimensions of a real trumpet. We employed two different solutions for the mouthpiece + cylindrical tube part and the bell. In the first case a cylindrical sections WGF proved to be accurate and simple enough. To carefully model the bell instead, we used a new technique in digital waveguide modeling, introduced by van Walstijn and Välimäki [7], which considers the horn function and the space dependence of the propagation constant.

Since the sound radiation mechanism is still unclear we chose to get the output pressure by direct filtering the mouthpiece pressure with a filter \( B(f) \) designed to fit a measured magnitude [2].

\[
\begin{align*}
p &= p_0 + p_i, \quad u = u_o + u_i \\
p_0 &= Z_c u_o, \quad p_i = -Z_c u_i
\end{align*}
\]

were substituted in (1), yielding

\[
p = \frac{2p_h - \sigma a(x)}{2} + \frac{\sigma}{2} \sqrt{a(x)\left[\alpha(x) + 4\sigma(p_0 - p_h)\right]}
\]

where

\[
\sigma = \text{sgn}(p_0 - p_h) , \quad \alpha(x) = \frac{2}{\rho} Z_c^2 A(x)^2 l(x)
\]

and in which \( Z_c = \rho c / S_{me} \) is the characteristic impedance of the mouthpiece entrance.

The resulting implementation scheme is depicted in figure 3, in which the numbers inside the blocks refer to the equations in the text.

5 Simulation results

Due to the number of parameters and their complex origin, it has been very difficult getting good evaluations on their value for each simulation. As initial values for geometrical dimensions and lip mass we used experimental results contained in [3] and [6], but for other parameters, such as equilibrium length and angle, \( l_0 \) and \( \theta_0 \) we decided to proceed by trials. In figs. 4 and 5 mouthpiece pressure and lip opening are plotted for the second and third modes of oscillation. Accordingly to previous measurements (see for example [3]) these typical ‘plateau’ pressure waveforms became smoother as the excited resonator natural frequency was increased. However, the primary purpose of this model was to include the duality of behavior of the lips as a valve. Experimental data suggest a behavior transition between the second and the third air column’s modes.

![Figure 3](image_url)

Figure 3. Complete block scheme of the simulated instrument.

Motion equations (3) and (6), were solved by using the 4th order Runge Kutta scheme; to relate fluidynamical quantities with lip motion the waveguide relations

\[
p_0 = Z_c u_o, \quad p_i = -Z_c u_i
\]

To achieve such a passage, we changed the ratio \( b/l \). In figure 4a we depicted the steady state trajectory of the lip for the second mode, corresponding to \( f_{10}=215\text{Hz} \), and the third mode, with \( f_{10}=352\text{Hz} \); the arrows indicate the motion direction while the embossed line corresponds to the time in which \( p \) is greater than 7/8 of its maximum.

![Figure 4](image_url)

Figure 4. Trajectory of the tip of the lip for the second (a) and third (b) resonance modes.
In the first case, acoustic pressure is applied after the maximum opening $M$, that is $G_01 > 0$; we therefore conclude that the second resonance mode supports the outward striking configuration.

In figure 4b, which refers to $f_{lip} = 352 \text{Hz}$, we have the opposite situation: $x$ reaches its maximum after $p$, so the predominant behavior is upward striking. A further confirmation of this transition comes from a comparison of the frequency relations in Table 2 with those in Table 1. Also, with particular choice of parameters, especially with high $p_0$, aperiodic oscillations were evident. Moreover, when $f_{lip}$ was between two impedance peaks, so that two modes were excited together, multiphonics could be generated. Such aperiodicities and multiphonics were also found by Adachi and Sato [1] on their one-dimensional models simulations.

**Table 2.** Comparison between $f_{lip}$, $f_p$, and $f_{sound}$ for second to seventh modes of resonator.

<table>
<thead>
<tr>
<th>Mode</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{lip}$ [Hz]</td>
<td>215</td>
<td>352</td>
<td>440</td>
<td>530</td>
<td>630</td>
<td>690</td>
</tr>
<tr>
<td>$f_p$ [Hz]</td>
<td>226</td>
<td>336</td>
<td>431</td>
<td>518</td>
<td>596</td>
<td>670</td>
</tr>
<tr>
<td>$f_{sound}$ [Hz]</td>
<td>232</td>
<td>319</td>
<td>401</td>
<td>476</td>
<td>565</td>
<td>595</td>
</tr>
</tbody>
</table>

**Figure 5:** Mouthpiece pressure and lip opening waveforms for the second resonance mode.

**Figure 6:** Mouthpiece pressure and lip opening waveforms for the third resonance mode.

**Table 2.** Comparison between $f_{lip}$, $f_p$, and $f_{sound}$ for second to seventh modes of resonator.

Conclusions

The two-dimensional model of lips proposed in this work coupled with a resonator based on the dimensions of a real instrument well reproduces many important typical features of a trumpet such as the ability to select one of the first resonances of air column by changing tension of lips, the ‘plateau’ shape of the mouthpiece pressure waveform, the quasi sinusoidal lip opening and the brightness increase in the attack transients.

However the main goal was the integration in the same model of two valve configurations, outward striking and upward striking, which together contributes to the acoustical behavior of the lips. Varying dimensions and mass of the lip we achieved a transition of the prevalent behavior which, accordingly to [8], is outward striking for the lowest modes and upward striking for the others. Although we achieved good qualitative results, we can’t be sure of the accuracy of quantitative evaluations, since exact values of many parameters weren’t available and we had to proceed by trials. Therefore our model, which proved its potential validity, has to be considered as preliminary to further researches.

**References**


