Super-Spherical Wave Simulation in Flaring Horns

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Abstract: The flared horn is modeled according to Webster’s equation. A change of variables transforms the equation into the form of the one dimensional Schrödinger wave equation. The Schrödinger form facilitates specification of arbitrary axisymmetric wavefronts for the acoustic disturbance within the horn. To provide a physically motivated choice of wavefront shape, Poisson’s equation is solved inside the horn subject to the boundary condition that the normal component of the potential gradient is zero at the boundary of the horn. Since the disturbance within the horn must satisfy the wave equation, the velocity potential satisfies Poisson’s equation when viscous effects and losses are ignored. Physical data from brass instrument horns are used to model musical horns using the Poisson solution, and results are compared to those obtained by traditional models which assume spherical wavefronts.

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1 Introduction

In [Berners and Smith, 1994] a method has been proposed to determine the acoustical properties of flared horns using a form of Webster’s horn equation developed in [Henade and Jansson, 1974]. The method allows any choice of acoustic wavefronts propagating in the horn as long as axial symmetry is preserved. Here we discuss an application of that method using Poisson’s Equation to predict the acoustic wavefronts within a horn.

2 Wavefront Computation

Because of similarities between the acoustic wave equation and the classical field equation for electrostatics, the isophase pressure wave surfaces at relatively low frequencies are equivalent to equipotential surfaces found within an insulator having the shape of the horn wall with an axial potential drop. An insulating boundary with a high relative dielectric constant causes equipotential planes to be perpendicular to the boundary. This is appropriate in comparison to the velocity potential within an acoustic horn, which also must have equipotential lines which are perpendicular to the boundary due to the fact that

$$\frac{\partial \Phi}{\partial n} = 0$$

at the boundary wall.
Wavefronts within a typical brass bell were computed using the electrostatic Poisson solver RE-LAX3D which was developed at the TRIUMF Meson Facility in Vancouver, Canada (triumf.a). The bell boundary was defined by sampled measurements taken with a bell mandrel. Figure 1a shows equipotential surfaces of the solved grid along with the horn boundary. The surfaces become less spherical in regions of greater flare. For a given assumed acoustic waveform shape, we define the equivalent radius as \( r(x) = \frac{\pi^2}{2} \sqrt{S(x)} \), where \( S(x) \) is the surface area of the wavefront crossing the horn axis at \( x \). Figure 1b shows equivalent radii for plane waves, spherical waves, and waveforms obtained by the Poisson solver within the test horn. It can be seen that the equivalent radius for the Poisson solution falls between those for the planar and spherical waveforms.

Figure 2a shows the barrier functions for the spherical and Poisson solution wavefronts. As defined in [Berners and Smith, 1994], the barrier is the normalized second derivative of the equivalent radius, \( \frac{d^2 r}{dx^2} \). The spherical wavefront model produces a higher, narrower barrier which results in a slightly higher cutoff frequency in the reflection coefficient shown in Figure 2b. However, the difference between the two models is minimal in terms of response, and it is likely that the spherical model would be acceptable for this test case.

References


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