State-Space Models: Virtual World for Composition

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ABSTRACT

This paper presents sonifications of linear state-space mathematical models commonly used in control engineering as a source for electroacoustic music composition. These models can represent dynamical physical systems with the advantage of providing a states vector. In this paper the contents of that vector is generated and sonified in real time. This allows the composer to experiment with timbre, gesture and texture through manipulating the models input. The paper presents two specific systems implemented in Max/MSP and SuperCollider: an inverted pendulum and a spring-mass-damper. Examples of their use in multichannel sound synthesis and composition are provided.

1. INTRODUCTION

Auditory displays and sonifications have gained attention as alternatives for representing data and information. The accuracy and simplicity of aural representations is of great significance, but the aesthetic aspect has also become more and more important [1]. And in recent years sonifications have been intentionally used as sound sources for composition [2].

State-space models are commonly used in control engineering for system modeling, controller design and simulation, as they provide a compact way for representing and monitoring physical systems. These models can be adapted to represent variables of interest. Applying the same mathematical principles, they can be used to shape and control sound. The paper presents the implementation of two state-space models in Max/MSP and SuperCollider: an inverted pendulum and a spring-mass-damper system, as well as their use as an interactive tool for sonification and electroacoustic composition.

1.1 General Description

State-space models describe physical systems through n differences equations grouped in a vector-matrix equation. The model consists of a set of inputs, outputs and state variables expressed as vectors. The dynamics of the system can be described as a function of the state vector and the input signal. In other words, it is possible to determine the behavior and evolution of a system at any time $t > 0$ if the value of the state vector and the input at that time are known [3].

A general state-space representation of a continuous linear time-invariant system with $m$ inputs, $p$ outputs and $n$ state variables is written in the form:

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}$$

(1)

Where $x(t) \in \mathbb{R}^n$ is the $n$-dimensional state vector, $u(t) \in \mathbb{R}^m$ is the $m$-dimensional input vector, and $y(t) \in \mathbb{R}^p$ is the $p$-dimensional output vector [4]. $A$, $B$, $C$ and $D$, are constant matrixes defined by the system parameters. A digital version of the continuous representation is obtained by sampling the system (1) and has the form:

$$\begin{align*}
x(k+1) &= \Phi x(k) + \Gamma u(k) \\
y(k) &= Cx(k) + Du(k)
\end{align*}$$

(2)

Where $\Phi = e^{A T_s}$ and $\int_0^{T_s} e^{A s} ds$ are obtained by considering a zero order sample and hold circuit with a sampling period $T_s$ [5].

2. IMPLEMENTATION

The implemented models have the form (2). For SuperCollider the class "States" was written to handle linear algebra operations; it takes as arguments any $\Phi$, $\Gamma$, $C$, $D$, and $u$. The output is the states vector, which is updated in real time according to the input value $u$, and a sample period $T_s$. For Max/MSP, different JavaScript objects were developed. Each object contains the specific matrixes $\Phi$, $\Gamma$, $C$, and $D$ for a model. They can take any numerical Max/MSP signal as input. The output state vector is updated in real time according to the input $u$ and the sample period $T_s$.

2.1 Inverted Pendulum

The system consists of an inverted pendulum coupled to the top of mobile cart as depicted in [6]. The input represents a desired new position for the cart. The aim is that the inverted pendulum reaches the vertical position once the cart arrives to the new position [7].

To validate the model the step response is used. This means applying a signal from 0 to a constant amplitude, to test the response to an abrupt change. The system outputs are cart position and pendulum angle. Figure 1(a) shows the step response
of the inverted pendulum simulated in MATLAB when applying a step input of 0.2 amplitude\footnote{Meaning a change in position from 0 to 20 cm.}. Figure 1(b) shows the step response of the model implemented in SuperCollider. In both figures, the cart arrives to the desired position and the angle is 0, which means pendulum has reached the vertical position. The same validation for this model was applied to the implementations for Max/MSP although for simplicity it is not shown in this paper.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1a.png}
\caption{Inverted pendulum step response. (a) MATLAB Simulation, (b) SuperCollider implementation.}
\end{figure}

\section*{3. SSM AND SONIFICATION}

State-space models provide a states vector. The variables in this vector represent the state-space of the system. For instance, the state-space of the inverted pendulum consists on the variables cart position, cart velocity, pendulum angle, and pendulums angular velocity. These four variables will evolve over time accordingly when an input is applied.

Figure 3 shows the evolution of the state variables for the inverted pendulum for the step input of 0.2 amplitude. If the input remains static, the states eventually reach a stationary state. If sonificaton is used to represent the system states instead of a graph, it is possible to have up to 4 variables to be mapped simultaneously into sound parameters in real time. The aim of the sonification is to find musical meaningful mappings rather than merely representing the models behaviour. The states can be mapped either into oscillators control rate parameters to generate sound synthesis, or sound transformation parameters such as time delay, granular synthesis, etc. The models can be excited in real time, slowed down, accelerated or frozen by manipulating the sample period $T_s$. This implies that certain timbres can be frozen if desired. Slowly changing the input $u$ can create textures, mak-
ing abrupt changes in the input signal can generate gestures, and simply letting the models arrive to their stationary state can create smooth musical transitions.

4. STATE-SPACE MODELS IN COMPOSITION

4.1 Sonifying SSM

Following are a few examples about possible ways of sonifying state-space models and their use in composition.

4.2 Inverted pendulum

Once the output vector states are available in real time and dependant on an input signal, it is possible to make sonifications in real time. For example, a two dimensional wave object in Max/MSP is used to create synthesis. The 4 output states of a virtual pendulum are mapped simultaneously into 4 input parameters of the 2d.wave object as shown in Table (1). The composition process then consists on choosing meaningful scaling values for the system states and planing the sonic outcome depending on these values and the system behaviour.

<table>
<thead>
<tr>
<th>State</th>
<th>scaled output</th>
<th>2dwave Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>position</td>
<td>150 - 3750</td>
<td>start point</td>
</tr>
<tr>
<td>velocity</td>
<td>0.01 - 40</td>
<td>y phase</td>
</tr>
<tr>
<td>angle</td>
<td>1-8</td>
<td>x phase</td>
</tr>
<tr>
<td>angular velocity</td>
<td>25 - 200</td>
<td>No. rows</td>
</tr>
</tbody>
</table>

Table 1. Inverted pendulum sonification using Max/MSP.

The selected input range includes values between -4000 and 4000. The system input u can be manipulated either by typing values, using a slider object and/or using a midi controller. As the input changes, different timbre transformations can be heard and also, when the pendulum reaches its stationary state, the sound quality stops changing. If the input is constantly changing, the stationary state is not reached and the transformations continue. It is important noticing that the cart’s position determines the reading start point in the 2d.wave’s buffer. In other words reading start point is directly related to the the system input u. A recording of this sonification using a crotale recording as sound source can be found at 4.

A stereo sonification using SuperCollider and AM/FM modulation is presented in Figure 4. The system input u can be manipulated either by typing values and/or using a midi controller. Position is mapped to the parameter f1, which is scaled and used as the frequency of the sawtooth wave. Velocity is mapped to the parameter f2, scaled and used as a frequency for a sine wave. To give variety to the sound the scaling factors. The model is excited by freely choosing dynamic input values between 0.2 and 15000. More gestural sounds are obtained when the input jumps between extreme input values, with Tc of approximately 0.02 seconds, and more textural when the input varies smoothly.

As the model is excited with different values, different timbres are generated, more rhythmic for input signals below 100, and more textural for inputs above 400. In most cases it is possible to recognise the behaviour of the system itself; i.e. when applying a force, the mass bounces until finally reaching a stationary point. A short recording of this sonification can be found at 5.

The cart position determines the frequency (pitch quality) of the sound, whereas the velocity determines the timbre quality. When the scaled velocity is less than 20, the sound has a more percussive rhythmic character, more evident when close to 0. For higher velocity values the timbre changes and is no longer percussion like. Therefore if we desire a percussive rhythm quality, we have to let the pendulum arrive to the equilibrium position. The sound’s pitch will be proportionally related to the position value – the bigger this value, the higher the pitch. If we are not interested in rhythm then we should vary the input values and freeze the system when a satisfactory timbre is reached. In addition, sound evolves more or less smoothly between timbres depending on the value of Tc. A recording of this sonification can be found at 5.

imp3 = (rate1 (f1), freq1 (f2),cycles1 (f1))

vosim8= (rate8 (f1), freq8 (f2),cycles8 (f1))

vosim1= (rate1 (f1), freq1 (f2),cycles1 (f1))

Figure 4. Inverted pendulum stereo sonification example.

4.3 Mass-spring-damper

A sonification for the mass-spring damper system using a vosim synthesiser is shown in Figure 5. It consists of 8 carriers, one per channel. Each carrier is modulated by the same states, so the resulting sounds are related per channel. Each carrier, however, is slightly different form each other due to the scaling factors. The model is excited by freely choosing dynamic input values between 0.2 and 15000. More gestural sounds are obtained when the input jumps between extreme input values, with Tc of approximately 0.02 seconds, and more textural when the input varies smoothly.

As the model is excited with different values, different timbres are generated, more rhythmic for input signals below 100, and more textural for inputs above 400. In most cases it is possible to recognise the behaviour of the system itself; i.e. when applying a force, the mass bounces until finally reaching a stationary point. A short recording of this sonification can be found at 5.

5 https://www.escholar.manchester.ac.uk/uk-ac-man-scw:264532

4 https://www.escholar.manchester.ac.uk/uk-ac-man-scw:264499

5 https://www.escholar.manchester.ac.uk/uk-ac-man-scw:264532

“2 A gesture is therefore an energy – motion trajectory which excites the sounding body [...]” 9.

3 A new position has been reached and the pendulum has returned to the vertical position.

6 https://www.escholar.manchester.ac.uk/uk-ac-man-scw:264513
5. MUSIC EXAMPLE

Synthetic Springs is a fixed media 8 channel piece based on the sonified mass-spring-damper system model implemented in SuperCollider. The sound sources are derived from the vosim based sonification presented in Figure 4; a 4 channel expansion of the synthesiser of Figure 5 controlled with the mass spring damper system; a stereo FM sonification, and a stereo recording of piano string.

The composition process consisted on experimenting with the model sonifications to find satisfactory sound characteristics, recording and making selections to include in the piece. Figure 6 presents the sound material distribution in the piece. The 8 channel vosim sonification textural quality was used (1’10”, 3’ to 4’45”), and also its gestural behaviour (5’35” to 6’50”). The 4 channel AM/FM sonification provides a rhythmic quality (1’40” to 3’), and transitions between rhythmic quality and texture (6’50” to 8’46”). The stereo FM sonification and piano recording are more localised sound sources used to provide contrast in timbre.

The piece can be found at https://soundcloud.com/rosalia-soria/synthetic-springs-stereo.

Figure 6. Synthetic Springs sound material diagram.

For some sections specific channels were selected, i.e. stems of only 4 channels of the vosim synthesiser, or stereo pair of the 4 channel AM/FM synthesiser. Additionally, some sounds were spatially relocated or expanded following musical ideas. The piece also includes sounds not generated using model sonifications such as a 8 channel low frequency synthesiser that can work as cohesive material. A stereo reduction of this piece can be found at 7.

5.1 State-space models as interactive tool

SSM are suitable for interaction not only with SuperCollider or Max/MSP objects, but also external controllers such as midi, OSC or any kind of sensors. They can be excited and sonified in real time. As tested implementations are valid for models up to 4th order (4 states), any dynamic system with these characteristics can be represented in SuperCollider and Max/MSP, provided that matrices $\Phi$, $\Gamma$, $C$ and $D$ are known.

6. CONCLUSIONS

State-space models used in composition are a very powerful tool for creating sound synthesis and transformations. This implies that part of the compositional process relies on the meaningful mapping of state-space vectors. So, when these states are all mapped simultaneously into sound parameters, particular characters and behaviours are generated that would be difficult to obtain otherwise. One important aspect is the creation of multichannel sonifications, which explore textural and rhythmic effects involving the spatial factor. SSM can be seen as customisable instruments that can be played in real time. SSM not only represent physical systems, but also chemical, economical, etc., which means a great variety of objects that can used and virtually connected to create new sonifications. Future work includes experimenting with multi-input and non-linear state-space representations.

Acknowledgments

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7. REFERENCES


