A SPACE GRAMMAR FOR THE STOCHASTIC GENERATION
OF MULTI-DIMENSIONAL STRUCTURES

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INTRODUCTION

It is my intention to propose the seeds of a structuring scheme which appears to have considerable power and offer much scope for further development and research. As a composer, in my computer assisted compositions I have made use of stochastic techniques (Jones 1980) and the ideas developed in this paper arise out of that background, from the staple of compositional algorithms (Kenakis 1971, Koening 1978), following the guiding principle of generating material of maximum cohesion and variety from the minimum of constraints.

Most stochastic compositional exercises have always tended to follow a linear progression, where a process advances through time, sometimes with reference to what has gone before, sometimes not. The stochastic web grammars to be discussed can operate across many dimensions so that when applied in a musical/temporal context the parameters specifying simultaneously occurring events are intrinsically related to each other as well as to their temporal neighbours. All events are computed in a logical rather than time-sequential order in a hierarchy of self-embedded structures.

Following recent trends in computational practice, formal grammars have been suggested, and in some cases implemented, as the most efficient means of specifying and realising musical ideas (Roads 1979, Holtzman 1979). In the structures to be described the alphabet or vocabulary of the grammar is pruned down to an insignificant status and recursive production rules are defined so that a resulting syntax almost entirely determines the structure.

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To introduce what I have termed a space grammar I will consider its operation in one dimension and then extend the theory into higher dimensions.

**ONE-DIMENSIONAL SPACE GRAMMAR**

A simple stochastic grammar is defined consisting only of a single variable 'A' and a single terminal 'a'. A set \( R \) consisting of the following two production rules is defined.

\[
\begin{align*}
A & \rightarrow AA \\
A & \rightarrow a
\end{align*}
\]

A probability array \( P = (p_1, p_2) \) is applied over the set \( R \) to determine which production rules should be applied. The sum of \( p_1 \) and \( p_2 \) will be 1, and \( p_2 \) must be greater than \( p_1 \) to guarantee termination of a sequence of generations.

A possible derivation tree of a sequence generated by this grammar is given in fig 1. Obviously, a string of 'a's not so makes little structural sense. However, the syntactical structure responsible for generating the string may be preserved by applying the grammar to divide up the space, in this case a one-dimensional straight line, such that whenever rule 1 is applied, the space is split at the centre leaving two halves to be subdivided by further applications of the grammar, and whenever rule 2 is applied the splitting process will cease. Thus the syntactic structure of fig. 1 will divide the line as shown in fig. 2. This can be interpreted musically as a means of dividing up the time-space which will generate the rhythmic structure transcribed at the bottom of the figure.

Increasing the value of \( p_1 \) will cause generations to split to a greater depth resulting in a fast rhythm with many notes of short duration. Increasing the value of \( p_2 \) will produce the reverse effect, with more longer notes.

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TWO-DIMENSIONAL SPACE GRAMMAR

The grammar can be extended into two-dimensions by introducing an additional production rule. The symbol $/n$ will be introduced indicating a split in the nth dimension. Thus the three production rules are now:

$$\begin{align*}
A &= A / 1 A & (1) \\
A &= A / 2 A & (2) \\
A &= a & (3)
\end{align*}$$

If applied over the 2-dimensional plane, production rule 1 will divide the plane vertically, production rule 2 will divide the plane horizontally. It is necessary also to redefine the probability array $P = (P_1, P_2, P_3)$.

It is not convenient to construct a tree-diagram when operating such a grammar in two dimensions, but the following sequence demonstrates a method of representing a possible application of the grammar.

Stage 1: $A = A A A$ (rule 1)

Stage 2: $A = A A A$ (rules 2 and 1)

Stage 3: $A = A A A$ (rules 2, 3, 3, 1)

Stage 4: $A = A A A$ (etc.)

Stage 5: $A = A A A$ (etc.)

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When used to divide up the plane this sequence of productions will produce the result shown in fig. 3. If the probabilities in P are adjusted to increase $p_2$ and favour vertical divisions, structures such as those in fig. 4 will result. If $p_3$ is increased to favour horizontal divisions, then fig. 5 represents a typical result. When interpreted musically, in the time/pitch space, increasing $p_1$ will favour sequential activity where notes will generally be shorter and change more frequently, whereas increasing $p_2$ will favour simultaneous activity where generally longer notes will sound in continuously shifting chordal groupings.

By allotting a scale of pitches along the vertical axis, and time along the horizontal axis, the second pattern of fig. 4 has been transcribed into the form of the short piano fragment of fig. 6.

**ADDITIONAL PRODUCTION**

The grammar can be made more powerful by adding a **null production**. A $\rightarrow \lambda$, which will reserve an empty space in the structure without actually filling it. Adjusting the probability associated with the null production compared with the probability of the terminal production (number 3 in the set above) will vary the overall density of the resulting structure.

Such a grammar was used to generate note data to be input to the MUSC 5 sound synthesis program. The grammar adapts conveniently into a recursive procedure definition which is available in a language such as LISP or PASCAL.

```plaintext
PROCEDURE compose (start, finish, intensity);
REAL start, finish, intensity;
BEGIN
SWITCH production := vertical, horizontal,
sound, silence;
CASE production [end (4)]:
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vertical : mid = start + (finish - start)/2;
compose (start, mid, intensity);
compose (mid, finish, intensity);
GOTO silence;

horizontal : compose (start, finish, intensity/2);
compose (start, finish, intensity/2);
GOTO silence;

sound : note (start, finish, intensity);
silence;
END;

The procedure 'note' merely writes the appropriate MUSIC 5 data statement to 'play' a note with values of start, finish and intensity applying when it is called. For the purposes of this particular experiment the actual frequencies were chosen at random.

By repeatedly calling itself this short 'compose' procedure will generate an entire compositional structure. Considerable variety of output can be achieved by changing the probabilities to vary the horizontal/vertical ratio, the overall sound density and the depth of detail. In the version of the procedure listed above, all the probabilities are equal.

Fig. 7 is a graphic score representing two different sound structures generated by this procedure with the initial call:

Compose (1,10,1500);
The structures may be mapped into a musical form in the same way as was done for a two-dimensional grammar. The block in fig. 9 (a), for example, may be separated into a series of slices widthways along an arbitrary 'timbral plane' to produce the proto-score of fig. 10.

The examples which have been given have been kept simple for practical reasons, and for clarity, but there is no reason why generations should not be continued to a very detailed level and the parameter scales expanded to produce very large and complex structures.

Further sophistication in structural manipulation may be achieved by qualifying or changing the production probabilities in P to be dependent on generation depth, for example, or to change quite arbitrarily during a generation sequence. A universe of little changes can be generated by inverting $p_0$ and $p_1$. Bipolar antithetical structures such as that in fig. 11 can be generated by interfering with $p_1$, $p_2$ and $p_3$. This structure resembles the arrangement of cooling fins on a motorbike engine, or some audio amplifiers. Musically, it corresponds to a situation where a number of parts hold continuous lines whilst others interject with a series of block chords. Such a technique is popular in the choruses of Italian grand opera: in the Grand March from Verdi's Aida, for example.
THREE-DIMENSIONAL STAGE GRAMMAR

Adding a third production rule will give a total of five alternatives:

\[
\begin{align*}
A & \rightarrow A/A \\
A & \rightarrow \lambda/A \\
A & \rightarrow A/\lambda \\
A & \rightarrow \alpha \\
A & \rightarrow \gamma
\end{align*}
\]

A typical string generated with these production rules could be the following:

\[
(\alpha/2 (\gamma/\lambda \alpha))/1 ((\lambda/\gamma))_1 (\alpha/2 \lambda)
\]

which, by extending the previous operations, may be used to divide up a three-dimensional block by slicing in half width-ways when rule 1 is applied, in height when rule 2 is applied, and length-ways when rule 3 is applied. Thus the above string is equivalent to the block in fig. 8 (a). Chunks terminated by rule 4 are shaded, chunks generated by rule 5, the null production, are not. These empty chunks have been removed in fig. 8 (b) leaving only the block which has been generated by the grammar. Two additional blocks generated by this same grammar are given in fig. 9. Each block in this figure is the inverse of the other, where \(\alpha\)'s have been replaced by \(\beta\)'s and vice versa.

Varying the relation between the \(p_A\) and \(p_\lambda\) probabilities varies the density of the structure produced. If \(p_\lambda\) is very small a cheese-like structure will result containing a few holes; if \(p_\lambda\) roughly equal to \(p_A\) will produce a structure with a sponge-like quality; if \(p_\lambda\) is large then a few blocks will be left suspended in a largely empty space.
MULTI-DIMENSIONAL SPACE GRAMMAR

The basic concept of a space grammar can be extended to any number of dimensions with productions of the form:
\[ \text{N} \rightarrow \text{N}_1 \text{N}_2 \]

This is ideal, and indeed necessary, for sophisticated musical application. At least ten spatial dimensions may be identified: three dimensions to specify the physical spatial location of a sound, dimensions for pitch, intensity and time, and up to four or more dimensions for representing various timbral indexes.

It is difficult to imagine the physical relationships involved in such a large number of dimensions. Yet as a potentially powerful mathematician's tool, sound is one of the few areas where an attempt can be made to conceptualize multi-dimensional activity. Making use of an n-dimensional space grammar, which can easily cope with these difficulties, a compact yet enormously powerful computational procedure may be employed to generate complex but coherent structures.

The following outline shows the general form of a multi-dimensional space grammar procedure. Here, \( t_1 \) and \( u_1 \) are the lower and upper value limits respectively of each dimension.

**PROCEDURE** compose \((t_{11}, u_{11}, t_{12}, u_{12}, \ldots, t_{nN}, u_{nN})\);

**REAL** \(t_{11}, u_{11}, \ldots, t_{nN}, u_{nN}\);

**BEGIN**

Choose \((d_1, d_2, \ldots, d_N)\) sound, silence;

\[ d_1 : \text{mid} = t_{11} + (t_{12} - t_{11})/2 \]

\[ \text{compose} \left( t_{11}, \text{mid}, t_{12}, u_{12}, \ldots, t_{nN}, u_{nN} \right) ; \]

\[ \text{compose} \left( \text{mid}, u_{11}, \text{mid}, t_{12}, u_{12}, \ldots, t_{nN}, u_{nN} \right) ; \]

GOTO silence;

\[ d_2 : \text{mid} = t_{12} + (t_{13} - t_{12})/2 \]

\[ \text{compose} \left( t_{12}, t_{11}, \text{mid}, t_{13}, u_{13}, \ldots, t_{nN}, u_{nN} \right) ; \]

\[ \text{compose} \left( t_{12}, u_{11}, \text{mid}, u_{13}, t_{13}, u_{13}, \ldots, t_{nN}, u_{nN} \right) ; \]

GOTO silence;

\[ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \]

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\[ \text{\ldots\ldots\ldots} \]
\[ \text{mid} = \text{id} + \{\text{id} \text{, id}\}/2; \]
\[ \text{compose } \{\text{id}, \text{id}\}, \ldots, \{\text{id}, \text{mid}\}; \]
\[ \text{compose } \{\text{id}, \text{id}\}, \ldots, \{\text{mid}, \text{id}\}; \]
\[ \text{GOTO silence; } \]
\[ \text{sound: note}\{\text{id}, \text{id}\}, \ldots, \{\text{id}, \text{id}\}; \]
\[ \text{silence; } \]
\[ \text{END; } \]

An additional procedural parameter may be incorporated to monitor the depth of generations. The 'choose' procedure selects the appropriate production, making use of the \( P \) array. By changing \( P \)'s simple control probabilities, large amounts of material with a great variety of characters may be generated. Large scale macro-forms and micro-structures formed from timbral clusters are taken care of simultaneously by the one procedure which may operate to as great a depth of detail as is desirable or practical.

Further extensions of the skeleton space grammar would be possible by defining a larger set of variables, alternative terminal functions, and by making structural divisions at other than equal ratios, for example, at the Golden Section. This paper has merely scratched the surface of what is, as far as I am aware, an extensive and largely unmapped area of exploration.

"To vast is the scope that lies open to things far and wide without limit in any dimension."

\[ \text{Lutrelius, Book One.} \]
REFERENCES


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Figure 1

Derivation tree of a string generated from a one-dimensional space grammar
Figure 2
Spatial and rhythmic interpretation of the derivation tree
Figure 3

stages in a sequence of derivations when applying a two-dimensional space grammar.
Figure 4

The plane divided by a two-dimensional space grammar with a slight bias towards vertical divisions.
Figure 5
The plane divided by a two-dimensional space grammar with a slight bias towards horizontal divisions
Figure 6
A transcription of figure 4(b) in traditional notation
Figure 7
A pitch/time score of sound data generated by a two-dimension space grammar
Figure 8
Dividing a block using a three-dimensional space grammar.
Figure 9
A block and its inverse generated from a three-dimensional space group.
Figure 10
A partial musical transcription of the block in fig.4(a)
A possible bipolar, geometrical structure generated from a three-dimensional space grammar.