Resynthesis of Piano Strings Vibrations
Based on Physical Modeling

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Abstract

This paper presents a method to resynthesize the vibration of a piano string using a "physical" model, whose parameters are extracted from the analysis of real sounds. In order to modelize the behavior of a single string vibrating in two orthogonal directions, we design a model with two coupled elementary waveguide models. Such a model let us restitute in a realistic and controllable way both the beats and the double decay observed experimentally on the partials. We illustrate the relevancy of this approach by presenting results obtained on signals measured directly on a string by laser velocimetry.

1 Introduction

The synthesis of piano tones is undoubtedly still one of the most challenging sound modeling problems. Synthesis methods based solely on signal models (such as the ones used on most commercial electronic pianos) are not fully satisfactory because of the difficulty to link the musical gestures to the fine characteristics of the sound itself. From a physical point of view, this difficulty is due to the complexity of the instrument which contains thousands of interacting parts. Among these interactions, one can quote the non-linear interaction between the hammer and the strings [1], the interaction between the strings and the bridge [7], and the interaction between the bridge and the soundboard which radiates an acoustic field related to the actual perceived piano sound [3]. All these phenomena lead to complex sounds in which all the components of the piano interfere.

In this paper, we focus on the problem of the resynthesis of the vibration of a single piano string, by means of physical modeling. This work is actually part of a more general project to simulate the physical behavior of the whole instrument: as we shall see it can readily be extended to the case of multiple strings, and the effect of the hammer can separately be taken into account. Moreover, in the mid/high frequencies range considered here, the soundboard only acts as a linear filter, with a smooth enough frequency response: we can therefore consider that, up to a linear filtering of our signal, the vertical vibrations of our string (at the soundboard) represent the resulting sound. As opposed to full mechanical simulations [2], which are very accurate but can be extremely computationally expensive, we look for a synthesis model which only takes into account the most relevant physical phenomena from a perceptual point of view, but that may eventually run in real-time. Waveguide models [6] are known to satisfy such assumptions, but the estimation of the corresponding parameters from the analysis of real sounds is often difficult. Moreover, waveguide models cannot take into account subtle features of the sound such as the beats due to the interaction between orthogonal vibration modes ("polarizations") of the string. These interactions occur at the bridge level, and on a real piano this effect is emphasized by the presence of several strings tuned to a very close pitch. We shall see how to design a satisfying propagative model by coupling two waveguide models, and how the parameters of this model can be estimated from the analysis of real signals, allowing a realistic resynthesis of the original sound. We shall here restrain the discussion to the bi-polarization phenomenon, but our model can easily be extended to the case of multiple strings.

2 Waveguide models for monopolarized string

By assuming that the vibration of the string is polarized in only one direction, one can show that there exists a relation between the solution of the 1D waves equation in stiff and constrained strings and the output of a waveguide model [4]. This relation allows the estimation of the loop filter from the analysis of real sounds. In this case, the modulus of the filter is related to the decay of the partials and the phase of the filter to the dispersion in the media, due in our case to the finite stiffness of the string. For many instruments this technique leads to very realistic sounds and allows in addition...
to transform the sound according to the change of physical parameters. Nevertheless, this approach is not fully satisfactory in the case of a string tied to a mobile bridge. Actually, the bridge plays the role of a weak coupling of the modes in orthogonal directions. As a result, there is a slow transfer of energy between these two eigen-directions of vibrations, causing the string to vibrate in an apparent “precession” movement, a well known phenomena amongst piano manufacturers. This leads to either a slow (typically on time scales of a few seconds) modulation or a double decay [8] in the amplitude of the partials of the sound. These perceptually critical features cannot be reproduced using an elementary waveguide.

3 A propagative model for bi-polarized string

In order to take into account the vibration of a string in the two orthogonal directions, we have designed a propagative model by coupling two elementary waveguide models. Each of these waveguides simulates the vibration of the string in a given direction, namely the horizontal and vertical direction (in a grand piano setup). This idea is an extension and generalization of the coupling proposed by Karjalainen et al. [5], where different strings are modelized by waveguides coupled by a constant real gain. This model generates the same behavior (double decay and modulations) for all partials, whereas as we shall see, our model let each partial exhibit a different behavior.

We here go one step above in complexity, by assuming that the couplings occur at both ends of the delay lines and are described by a complex frequency dependant gain. This propagative model is represented in figure 1.

\[ T_1(\omega) = \frac{P(\omega)e^{-i\omega D}}{1 - X(\omega)e^{-i\omega D}} + \frac{Q(\omega)e^{-i\omega D}}{1 - Y(\omega)e^{-i\omega D}} \]

\[ T_2(\omega) = \frac{R(\omega)e^{-i\omega D}}{1 - X(\omega)e^{-i\omega D}} + \frac{S(\omega)e^{-i\omega D}}{1 - Y(\omega)e^{-i\omega D}} \]

where \( X(\omega), Y(\omega), P(\omega), Q(\omega), R(\omega), S(\omega) \) can easily be expressed as functions of \( F_1(\omega), F_2(\omega), H(\omega), F(\omega) \).

In the time domain, these doublets yield a sum of decreasing exponentials:

\[ T_1(t) = \Theta(t) \sum_{k=-\infty}^{k=\infty} a_k e^{-\alpha_k t} e^{i\omega_{1k} t} + b_k e^{-\beta_k t} e^{i\omega_{2k} t} \]

\[ T_2(t) = \Theta(t) \sum_{k=-\infty}^{k=\infty} c_k e^{-\alpha_k t} e^{i\omega_{1k} t} + d_k e^{-\beta_k t} e^{i\omega_{2k} t} \]

where \( \Theta(t) \) is Heavyside’s step function and \( \omega_{1k} \) and \( \omega_{2k} \) are the resonance frequencies of the two components of the \( k^{th} \) partial which are defined by:

\[ \omega_{1k} = \frac{2k\pi + \Phi_X(\omega_{1k})}{D} \quad \omega_{2k} = \frac{2k\pi + \Phi_Y(\omega_{2k})}{D} \]

\( \Phi_X(\omega) \) and \( \Phi_Y(\omega) \) being the phase of \( X \) and \( Y \), respectively.

The parameters of the “doublets” are related to the filters of the model by the following formula:

\[ \alpha_k = 1 - \frac{|X(\omega_{1k})|}{D|X(\omega_{1k})|} \quad \beta_k = 1 - \frac{|Y(\omega_{2k})|}{D|Y(\omega_{2k})|} \]

\[ a_k = \frac{P(\omega_{1k})}{DX(\omega_{1k})} \quad b_k = \frac{Q(\omega_{2k})}{DY(\omega_{2k})} \]

\[ c_k = \frac{-R(\omega_{1k})}{DX(\omega_{1k})} \quad d_k = \frac{S(\omega_{2k})}{DY(\omega_{2k})} \]
The corresponding inverse problem is of great interest for the resynthesis of sounds since it consists in estimating the parameters of the model (the filters) from the analysis of real signals. By assuming that the frequencies of the components of a doublet are close enough to write \( \omega_{1k} \approx \omega_{2k} = \omega_k \) (experiments showed that the frequencies are generally within less than 1Hz), and that the frequency behaviour of each filter is regular in the vicinity of the resonant peaks, one can derive both the phase and the modulus of the filters at the peaks:

\[
X(\omega_{1k}) = \frac{e^{i(\omega_{1k}D-2k\pi)}}{a_k D + 1}, Y(\omega_{2k}) = \frac{e^{i(\omega_{2k}D-2k\pi)}}{\beta_k D + 1}
\]

\[
F_2(\omega_k) = \frac{(a_k + b_k)X(\omega_{1k})Y(\omega_{2k})}{a_k X(\omega_{1k}) + b_k Y(\omega_{2k})}
\]

\[
F_1(\omega_k) = \frac{a_k X(\omega_{1k})^2 + b_k Y(\omega_{2k})^2}{a_k X(\omega_{1k}) + b_k Y(\omega_{2k})}
\]

\[
E(\omega_k) = \frac{D(a_k X(\omega_{1k}) + b_k Y(\omega_{2k}))^2}{a_k X(\omega_{1k})^2 + b_k Y(\omega_{2k})^2}
\]

\[
F(\omega_k) = \frac{a_k + b_k}{a_k + b_k}
\]

\[
H(\omega_k) = -\frac{a_k b_k (X(\omega_{1k}) - Y(\omega_{2k}))^2}{(a_k + b_k)(a_k X(\omega_{1k})^2 + b_k Y(\omega_{2k})^2)}
\]

Thus, by analysing the vibration produced by a string, and estimating the parameters describing the behaviour of each spectral component, one can fit the parameters of the model and resynthesize a signal similar to the original.

4 Results obtained on an experimental setup

To validate both our physical assumptions and the accuracy of the resynthesis model, we have designed an experimental setup to measure the string velocity in orthogonal directions at a single point (close to the bridge). The setup consists in a rigid and massive support on which the string is tightened with real piano hookings. By means of weights and nylon threads, we are able to pluck the string in a reproducible way. The string velocity is measured in orthogonal directions with a laser vibrometer pointing on the string. The extraction of the amplitudes, decay times and frequencies of the partials is performed through band pass filtering to isolate each partial, followed by a parametric modeling to separate the components within a same doublet.

As an example, figures 2 and 3 show the amplitude modulation law of the third partial on a 5 s time interval and its spectrum respectively for the vertical and horizontal polarization. The double resonance visible on the spectra introduces beats on the amplitude modulation laws. Notice that the frequencies of each doublet are the same in the horizontal and vertical polarization.

Fig 2: Third partial, vertical polarization
Top: Amplitude modulation law.
Bottom: Blow up of the spectrum

Fig 3: Third partial, horizontal polarization
Top: Amplitude modulation law.
Bottom: Blow up of the spectrum

Figures 4, 5 and 6 show respectively for the 8 first partials, as functions of the partial number, the modulus of the excitation signal \( E(\omega) \) corresponding to the pluck on the string, the modulus of the filters \( F_1(\omega) \) and \( F_2(\omega) \) describing the propagation in each direction, and the modulus of the coupling filters \( F(\omega) \) and \( H(\omega) \).

Fig 4: Spectrum of the excitation on the 8 first partials.

The excitation shows features related both to the spectrum of the excitation and to the locations of the excitation and the measurement points. This
is why the 7-nth partial is much weaker than the others (the plucking was performed at a position close to 1/7 of the string length).

![Graph](image1)

Fig 5: Spectrum of the propagation filters on the 8 first partials. Top: $F_1$, Bottom: $F_2$

The modulus of the propagation filters represents the damping of the string vibrations due to its own losses (internal losses and air friction), and also due to energy transfer to the bridge. As expected, it is very close to 1.

![Graph](image2)

Fig 6: Spectrum of the coupling filters on the 8 first partials. Top: $F$, Bottom: $H$

The coupling filter $H(\omega)$ shows a strong peak for the third partial, meaning that the two polarizations of the string are strongly coupled at this frequency. Actually, the frequency of the third partial corresponds to a resonance of the "bridge" of our experimental setup. This has been confirmed by a separate estimate of the admittance of the bridge.

By the use of a specifically designed experimentation, we showed that the filters of the model can be directly related to the admittance at the bridge level. Further investigations have shown that this model can be readily extended to the case of two coupled strings.

The model we proposed and the estimation techniques we used let us resynthesize perfectly the signals collected on our experiment. It can also easily allow sound transformations corresponding to physical modification on the string set. Among them, we can quote the detuning of the two strings and the production of the equivalent of sympathetic resonances on a real piano. We worked on signals from a specific experiment but the model in itself remains valid in the case of real piano tones. For musical applications, this model would let us, for example, simulate the replacement of one of the piano string by a nylon guitar string and would of course let us play in real time with this virtual instrument.

References


