Protoyping and Interpolation of Multiple Musical Timbres
Using Principal Component-Based Synthesis

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ABSTRACT

Principal Component Analysis (PCA) has been shown to be an effective technique for representing the complexity of time-variant musical tones efficiently. We explore here a novel application of PCA to Phase Vocoder Analysis (PVA) datasets that yields both common and distinct patterns of spectral evolution for groups of instrument tones. A prototypical timbre is synthesized for the group from the spectrtemporal patterns common to the group's members. Interpolation is accomplished by applying to the prototype the spectrtemporal patterns that distinguish between members. We also introduce a variable-duration temporal partitioning technique for preprocessing the PVA datasets that resamples their size by roughly a factor of 8 while retaining high temporal resolution for perceptually critical portions (attack and decay). This downsampling improves the sensitivity of the PVA to important details of the PVA data, and produces more natural-sounding attack and decay transients when resynthesized at a variety of durations.

1. INTRODUCTION

Many researchers have explored data reduction techniques for efficient reduction of additive synthesis data sets without loss of perceptually important detail (see Laughlin et al., 1990, for a list of sources). Principal Component Analysis (PCA) along with the related Karhunen-Loève Transform appears to be an effective technique for reducing non-spectral structures (Laughlin et al., 1990; Stapleton and Buss, 1985), suggest possibilities for real-time control (Weeks et al., 1991), and perform efficient identification (Spitzer, 1983). In this paper, we report on three new developments that improve Phase Vocoder Analysis (PVA) data reduction both qualitatively and quantitatively. The first development is a variable-duration temporal partitioning, (VDTP) of the PVA datasets that emphasizes attack and decay transients. The second development is the application of PCA to spectral profiles across time, in contrast to the approach of Laughlin et al. (1990) which applied PCA to temporal amplitude envelopes across partials. The third development addresses the question of how interpolations between various timbres may be obtained, using PCA in combination with other multivariate statistical techniques.

2. DOWNSAMPLING

An important reason for our downsampling the PVA datasets before submitting them to PCA is to emphasize the most perceptually critical portions. While the steady-state portions of instrument tones can survive a significant downsampling in temporal precision, the attack and decay transients cannot. Therefore, we have broken the timescopes of each tone into 200 partitions of variable duration. For our PVA datasets containing a total of 1200 to 1800 analysis time frames, roughly 80 of the 200 partitions are dedicated to capturing attack transients at the original PVA frame rate. During the relatively slowly-varying back of the tone, partitions durations gradually increase to a maximum of roughly 24 frames, and then decrease towards the end of the tone in order to capture the decay at the original frame rate (2.14 ms per frame). Fig. 1 shows a typical distribution of partition durations. In 1a the number of PVA time frames subtended in each of the 200 partitions are shown; in 1b, the varying duration of the partitions is shown with respect to the original 1508 PVA time frames.

In order to shift a dataset back to its original temporal resolution, a spline is used on each harmonic amplitude envelope. Nearly half of the partition values are unaffected by the resampling since they correspond to single analysis frames. The detail that was lost in the process of downsampling and resampling had virtually no audible consequence for any of the sounds we investigated (wind instrument tones from the McGill University Master Samples and ProGenus CDs). Although steady-state portions were somewhat smoother than their original analysis as a consequence of the splineing technique, this did not seem to injure the natural quality of the tone. Furthermore, the whole process runs without any user intervention, in contrast to the segmentation required for quality Karhunen-Loève synthesis (Spitzer and Buss, 1985).
Since attacks and decays are not significantly compressed in downsampling, they sound natural even when a 4-second tone is shortened to 0.5 seconds and vice-versa. This complements the popular time expansion and contraction effects afforded by the phase vocoder that rescale time without changing pitch or spectral energy distribution. And since we regenerate each harmonic amplitude envelope independently, spectral evolution continues throughout the course of the tone, albeit quite slowly during the steady-state portion of a time-expanded tone.

One of the major advantages of VDTIP is that it maps times of unequal duration into a common-resolution non-linear time domain. In effect, we have overcome the obstacle of finding the constant number of temporal variances required for analyzing multiple tones in a single PCA, making possible a novel application of PCA to PVA datasets. As the next section of our paper explains, and in contrast to Laughlin et al. (1990), we submit the transpose of the matrix of PVA data to PCA, a technique we call spectral PCA.

3. SPECTRAL PRINCIPAL COMPONENTS ANALYSIS

We describe PCA here informally as it relates to the context of additive synthesis data reduction. For more formal detail, the reader is referred to Harris (1985) and Marsico (1987). Although our exposition here refers to the manipulation of amplitude information only, our work includes equal treatment of both the amplitude and frequency information from PVA datasets.

PVA datasets of instrument timbres contain inherent redundancy since the temporal amplitude envelopes of partials in natural instrument sounds are correlated, as in Fig. 2. This redundancy can be reduced to just a few dimensions of orthogonal basis vectors; for example, most of the activity in harmonics 4-12, with their early sharp peak, can be captured by one basis vector, while the slower-rising, flat-peaked shape of harmonics 1-3 can be captured by another. Successive principal components (PCs) add features to account for the idiosyncratic features of one or more other harmonics (such as the late "bumps" in partials 11 and 13) and/or as a corrective term for features that must be removed from certain harmonic amplitude envelopes. In this approach, the one adopted by Laughlin et al. (1990), a PVA dataset is input to PCA as a series of temporal envelopes, one for each harmonic; consequently the basis vectors themselves resemble temporal envelopes.

We refer to this approach as temporal PCA. An alternative view of a PVA dataset is as a series of spectral envelopes for each time frame, as shown in Fig. 3 (here the entire tone is shown in only 20 frames). Each of the four basis vectors shown (top row) resembles a spectral envelope, and each successive component adds more of the detail that distinguishes the spectral envelope at each point in time. We refer to this as spectral PCA.

To make the distinction between these two methods clear, we must examine the orientation of the matrix input to PCA: If the downsampled PVA data matrix is organized with rows corresponding to frequency values, and columns corresponding to time values, then we have spectral PCA. This analysis capitalizes on correlations between the columns of spectral envelopes. If the PVA data matrix is transposed, we have temporal PCA that capitalizes on correlations between the columns of temporal envelopes. Note that without the VDTIP downsampling, spectral PCA on the original PVA data would require the computation of a huge covariance matrix (1508 by 1508 for the trombone example shown in Fig. 1). The rank of the covariance matrix for temporal PCA is determined by the number of harmonics (22 in our examples).

Both methods result in an efficient reduction of the data by the determination of orthogonal vectors in the dataset. An exact recovery of the original dataset is guaranteed by using all PCs. However, when using only the 3 PCs we found...
necessary to capture the perceptually relevant variance, the result was data reduction greater than a factor of 67 in our examples. Moreover, PCA can reveal high-level features that may be perceptually relevant and/or useful for musical manipulation, but which are not apparent by viewing the raw data.

We prefer spectral PCA to temporal PCA for our exploration of timbre for several reasons. As a rule, PCA is an efficient data reduction method only when the input variables are correlated. In natural sounds, the spectral envelope changes smoothly through most of the tone's duration, meaning that there is a great deal of correlation between neighboring time frames. Temporal envelopes, on the other hand, are often less highly correlated (e.g., flute and violin). In fact, all the tones employed in our study showed a higher average correlation of spectra over time frames than correlation of temporal envelopes over harmonics. For example, the resynthesized spectra shown in Fig. 3 resembles the original data in significant detail by the 3rd PC, whereas a resynthesis was temporal PCA (not shown) needed at least 6 PCs to reach the same degree of detail. Admittedly, temporal PCA might be more efficient for tones with a great amount of spectrally temporal flux, but for the type of tones we investigated, we found spectral PCA most efficient.

![Figure 2](image1)

![Figure 3](image2)

Furthermore, spectral PCA is more space efficient when analyzing multiple instrument tones. This is because the basis vectors are usually longer in temporal as compared to spectral PCA. Though the weighting functions are perform longer in spectral PCA, these are common to all instruments, so as the number of instruments grows large, the amounts of data required for temporal vs. spectra PCA depends only upon the length of the basis vectors. In an example using 3 PCs (and PVA datasets of 22 frequency values by 200 time values), we found that reduction roughly 10 times greater using spectral PCA. This obviously has important ramifications for synthesis using real-time hardware since spectral PCA can store information on spectral evolution for many more instruments in the same amount of memory. Experiments for real-time synthesis on the Silicon Graphics Indigo are underway using techniques described in Freed (1990) and Rodes and Depalle (1992).

From a more theoretical perspective, we posit that a characterization of spectral differences over time may be more useful than information on the envelope differences across harmonics. For example, consider a PCA of several tones from the same instrument (different pitches, articulations). Since spectral properties are more likely to be invariant across multiple durations and articulations of an instrument than temporal properties, a single spectral PCA may offer more insight into an instrument’s general timbral identity.

4. PROTOTYPING AND INTERPOLATION

We have described a mechanism for packaging tones of unequal duration into downsampled PVA data matrices of 22 frequency values by 200 time values. We have also shown that these data can be simplified further through the
application of spectral PCA. Now we show the advantage of breaking that spectral PCA into two parts, employing methods from Multivariate Analysis of Variance (MANOVA; see Harris, 1981). The first part is the generation of a prototypical timetable from a set of timbres using a PCA that finds weighting functions only for what distinguishes between the timbres. The prototype weighting functions were generated by finding the eigenvectors of the pooled within-groups sums of squares and crossproducts (SSCP) matrix. The interpolation weighting functions were based upon the between-groups SSCP matrix. The within-group and between-group SSCP matrices sum to equal the total SSCP matrix for all the PVA datasets submitted, hence the information for complete reconstruction of the original datasets is not lost in these operations.

By isolating what all the tones have in common, we can generate a prototypical tone that captures none of the features of any tone in particular. When several different horns were analyzed, the resulting prototype sounded like a band, generic horn sound. This result is perhaps not musically useful, but if we subtract the prototype horn from all the individual instrument matrices, the prototype becomes the origin of a multidimensional coordinate system within which each instrument is located at a unique point relative to that origin. This has the musically useful result of setting up a control structure for timbral interpolation. To reduce the dimensionality of the interpolation coordinate system, the deviation matrices were scored on the interpolation weighting functions. Using only the first 3 PCs for the deviation scores creates a coordinate system that can be easily explored.

The primary source of information for synthesis is in the PCs for deviation scores. Once the prototype is generated, it becomes a "center of gravity" for the space within which interpolation takes place. Note that a prototypical timbre could have been synthesized from a simple average of all the PVA data matrices in the set of timbres, but we found that this result had objectionable idiosyncrasies that did not appear in the more idealized PC-based prototype. Since the interpolation space is defined by 3 PCs, there are multiple paths between each of the analyzed timbres. We can take the shortest path between two timbres, or follow a piecewise linear path that changes values on one dimension at a time.

5. CONCLUSION

We have reduced and regularized the data of phase vocoder analysis through VDTP down-sampling (reduction factor of roughly 8), and then reduced it further via spectral PCA. The fixed-memory-size cost of PC-based synthesis is 280 points per PC weighting function, regardless of the number of timbres represented. If 8 PCs are employed, then 200 PVA frames can be reduced to 3 basis vectors (a reduction factor of 67 for each timbre). Ignoring the fixed-size weighting function matrix, the combination of these two procedures gives a reduction factor of more than 500 with respect to the original PVA data. We see that the data reduction provided by VDTP down-sampling is not nearly as great as that obtained by fixing line-segment approximations to each amplitude envelope (see Grey, 1975), but such approximations are not suitable for PCA, and do not even possess many of the desirable features of VDTP down-sampling.

There are several advantages offered by these methods for compositional exploitation of timbre. Spectral PCA allows us to keep spectral attributes of an analyzed timbre intact while exploring creative temporal manipulations. Furthermore, the potential for timbre interpolation as described in Wessel et al (1987) and Grey (1975) is offered by our method of representing multiple tones.

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7. REFERENCES