PHYSICAL MODELS OF INSTRUMENTS
A MODULAR APPROACH, APPLICATION TO STRINGS
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Abstract
This paper describes our work on the simulation of physical
instruments, including a resonator, a string, and entering
like lutes, harpsichords, plucked or fingers, as well as our effort
toward a modular conception aimed at a better use and
enhancement of the simulation programs.

1. Introduction
The essential characteristics of sympathy by physical models
are that it offers a pre-organized material, and that physical
rules which are inspired in normal vibrating motion produce
such structuring.
The pre-organization of the material in today's widely
considered as necessary because the raw playing sample
does not in every case stimulate the computer sufficiently,
and may produce unexpected musical results. Therefore, the
creative energy of our composers is often exhausted befocuse
when this pre-organization is useful. It offers the composer
an environment of a virtual structured objects. Physical-model synthesis is in fact
with respect to the first level of organization an original option in
which elementary structures are made with mechanical and
acoustic rules.
Physical rules produce objects of the physical space, and,
the combination of these objects makes sounds which are
strongly bound to their natural conditions, the physical
description given, for a sound, a reference to the producing
people. Our work takes place in the same domain much more
than in frequency domain, in other words, the physical-
model synthesis is more concerned with the cause than with
the effect. The problem is that the first level of structure has
to be wide and general enough to be used in a musical
context, and precise enough to provide the objects with the
acoustic features; one might regret, in this respect,
the stiffness of physical rules.
We expect that this environment will stimulate the
composer in the realm of structure, because the bonds
between elementary objects produced by the same cause
are useful tool in constructing musical terms, and in their
realm of expression as well because the perception
mechanism associate a given effect with a virtual same-
white characteristic are determined by the perception
features of the effect. In many cases the subjective power,
the weight of the object is closely correlated to the physical
energy modeled in the production.
Our sources for strings arise from the work of Smith
[Smith67] on the violin, Wever [Wever86] and
Boughien [Boughien79] on the piano. We must refer as well to
the work of Cadeau [Cadeau79] on physical model
synthesis.

2. Resonator
The direct attack for describing the string is to represent
it as a series of spring and masses [Cadeau79] [Cadeau79]
We know that in good approximation such a physical system
behaves like the physical mass continuum of a string. The
integration of the equation of dynamics for each mass and
each time step gives the position of the masses. It is
possible, then, to introduce losses of energy in the string, as
well as string stiffness etc. The method is physically very
satisfying but its drawback is that it implies the calculating,
each time step, of the position of each mass, whereas
only the masses concerned by the excitation, and the
masses close to the bridge are really significant in a musical
context, because they are highly responsible for
the produced sound.
Another method consists in the representation of the
described by a double finite line, each line supporting a wave
propagating in the opposite direction, and reflected at the
each of the line into the line other. A single delay line as the
entire element of the Kaykin-Strong algorithm [Kaykin89]
and as well as the Wilke model of Smith, both approaches
involve even signal processing techniques than proper
physical description, and were consequently limited in terms of
simulations. We adapted some of Smith's solutions to our
resonator, being always eager to preserve the physical
significance of the model.
The movements involved are unidimensional, and the
physical method chosen for calculating the shape of the free
oscillating string at each time step is a method of

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ICOM '85 Proceedings
The significant points for the user are the last cells of the delay lines at the bridge. These two cells are in fact the output and input of the string. Output is sound, and input is usually information coming from other strings through the bridge. As a result, for the user, the string is an input/output module, provided with a dynamic control system (fie). Note that the integration by arbitrary functions does not imply any discontinuities of the mass strings at the string, and that the string m in our model, a continuum.

3. Illustration

The general mechanical system used is model figures, phyla, harpers, and hammers or tamper represented in Fig.2. The mass m is in contact with one point of the string and the operator applies a force F at the mass M. The two masses are bound with a spring k and we have included losses B and C. In the case of a harp, the masses M and m may represent the mass of the neck and of the feather which plucks the string; for the piano, they are the masses of the hammer and of the felt, and one can easily imagine in the case of the grinder, the same kind of mechanical model, where B represent the stiffness of the pul.

Figure 1

Mechanical model for the exciter

Two masses m and M, a spring k and resistances B and C. The mass m is in contact with an string, and the operator applies a force F at mass M.

During the excitation, we consider that the signals leaving and arriving at the point of excitation are the incident transmitted and reflected waves from each side of the point. If we consider then that u(t), v(t), x(t) and y(t), the waves on the resonator and each side of the mass m, and the absolute expression of the waves applied by the string to the mass m, and the set of differential equations for the absolute x(t) and y(t) of the masses M and m, where x(t) and y(t) are the instantaneous inputs at the point of excitation. Integration of this set of equations gives at each time step the values of Y(t), y(t), y(t) and y(t), as a function of the instantaneous inputs x(t) and y(t).
Further, we find it interesting to have a simple onmass model for the excitation. It is actually identical with the two-mass model provided with a peak of infinite stiffness. For a given sampling rate, the quotient of mechanical stiffness by mass is limited in the same way as frequency. The set of differential equations reduces then to a single equation describing the movement of the single mass $m$ in contact with the spring:

$$\frac{d^2x}{dt^2} + 2\zeta\frac{dx}{dt} + \omega^2x = \frac{F(t)}{m}$$

For the bow, we have adopted the well-known functional description using the static friction characteristic $f(x, v)$ and describing the motion of the bowed string as a free oscillating motion. The expression of the force $F(t)$ applies from the string to the bow and of the string's contour at the point of contact lead to the set of equations giving the outputs $y_1(t)$ and $y_2(t)$ as a function of the instantaneous inputs $x_1(t)$ and $x_2(t)$:

$$y_1(t) = \frac{f(x_1, \frac{dx_1}{dt})}{m_1}$$
$$y_2(t) = \frac{f(x_2, \frac{dx_2}{dt})}{m_2}$$

In all the cases, we may represent the exciter as a double input/output module (Fig. 2) which computes the instantaneous outputs as a function of instantaneous inputs at the point of excitation, and which is not concerned with anything else happening on the string. This representation has two advantages: The first point is that the exciter takes into account all the past information present on the string as it begins to act. In other words, we may play two identical notes on either end in a natural way, which was not the case in previous models. The second point is that we may have many excitations on the same string at the same time, each one of them computing as if it were alone, i.e., rather independently from the others, and this is a good help in modular conception. Besides, two pairs of allpass filters allow the exciter to act at any place on the string, and to move dynamically during the excitation.

For the plucked string, Fig. 3 shows the movement of the spectrum during the excitation. The dotted straight line is the position of the string at rest. Dashed and solid curves are the trajectories of the masses $M$ and $m$. From the contact to the moment of release, the mass $m$ pulls the string away from its equilibrium position, and the sound produced in this time interval is a good approximation of the initial attack sound. Time of release is determined with a condition $F < F_{max}$ on the force applied by the string to the plucked spectrum. After release, the masses oscillate around $M$. We have chosen a reasonable distance in order to make evident that the shape of the string at release is, in the case of a dynamic attack, quite different from the well-known triangle shaped string of ideal plucking. Fig. 4a shows the shape of the string after release and Fig. 4b the same string after a twelfth of the fundamental period.

![Figure 3: Trajectory of the plucked string.](image)

![Figure 4: Shape of the plucked string.](image)
For the hammer, the trajectory of the contact point (fig. 7) (solid curve) shows the effect of the first and second reflected waves on the bridge, which try to push the hammer away. They do not achieve this, in fact, and we have to wait until the third reflected wave. At this moment, the hammer has lost its energy and is pushed away by the string. This appears clearly in the plot of the force exerted by the string to the hammer during the interaction (fig. 8). On the right side, a graph is displayed by Boulton (Boulton and White), and on the left, results given by the model. In the modelled situation, you can see at the end of the interaction a second interaction. This means that after the first release, the hammer which has lost its energy is moving slowly, and the string which is better pushes it away a second time. In any case, the second interaction is not very important with respect to the first. And it cannot be seen either in the experimental measurement or in the model trajectories. The corresponding shapes of the string are given in fig. 6 (a,b,c,d) for the first contact, first release, second contact and second release. Further measurements by Boulton helped us in the adjustment of the mechanical parameters.

Figure 5
Acceleration of the hammer; second interaction produced by the third reflected waves after the first release.

Figure 6
Shape of the hammered string:
fig. a,b,c,d for the first contact, first release, second contact and second release.

We have included modules representing loads of different kinds (masses, springs, beams) for the string, in order to provide the model with a wider range of inharmonic sounds. The mechanical properties and the position of the loads can vary dynamically (Adriaenss). For the finger, the different mechanical parameters are adjusted by ear and with patience, and the contact with the fingerboard is obtained in giving an important value to the mass m. At this moment, the finger, the string and the instrument are in contact, and the string is actually loaded with an important mass at this point. We considered that the damper of the piano, responsible for the noise cut off, is a very soft and absorbing hammer pushed against the string. The bow model gives the well known results of the Helmholtz motion. We are not yet satisfied with its musical qualities, and intend to model a more realistic case in order to account for sticking and sliding friction (Cremers). This is possible within our double input/output representation of the excitation.

5. Conclusion

We have concentrated our work on excitation, and we wish to investigate further the properties of the resonator, in order to obtain, for example, a more realistic bridge or stiff string. In every situation, a compromise between physical realism and computation costs has to be found. The synthesizer is then to be scheduled in a process-oriented programming environment (Concerto) at Ansermet. We expect that high level control of these physical objects will produce interesting situations in a musical context.
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