ON MAPPING N ARTICULATION 
ONTO M SYNTHESIZER-CONTROL PARAMETERS

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ABSTRACT: A technique for providing an arbitrary mapping from performance to control parameters is described: N articulation parameters define an N-dimensional space, at points in which are stored M control parameters. In order to allow for several performance parameters, the position of defined points is coarsely quantised. An interpolation scheme is described which ensures first-order continuity and allows a conventional ordinate mapping, yet only has the computational complexity of a simplex partitioning of the space, i.e. O(M·N) compared to O(M·2^N).

INTRODUCTION

The degree of control which a performer can exert over computer-generated sound is very often rather limited compared to that possible with conventional acoustic synthesizers. Sometimes a limited technique such as FM is employed for reasons of computational efficiency, and performance controls are mapped one-to-one onto synthesizer controls. The changes in timbre produced by changes in articulation with such a scheme are often rather crude when compared with those of a good acoustic instrument. Recent advances have made the real-time implementation of general synthesis techniques such as additive synthesis a practical possibility. Such synthesizers allow potentially unlimited control over timbre, but in practice are impossible to control directly in real-time because of the large number of control parameters that need to be continuously updated. In order to control such systems effectively, it is necessary to perform a mapping from the N measured articulation parameters onto the M parameters that control the synthesis at the lowest level.

The N articulation parameters derived from a performance can be used as orthogonal basis vectors defining a hyperspace of N dimensions [Bowler 1985]. If articulation parameter i is quantized to ni values, the number of points defined by the N parameters is

\[ U = \prod_{i=1}^{N} n_i \]  

(1)

It often not possible to store a set of M synthesizer control parameters at each of these points because storage requirements would exceed those available in current computer systems. A solution to this problem is to store the control parameters only at more coarsely quantised positions in the space, giving a smaller set of points, Q. Each point in the space then lies within a measure polytope (a squashed hypercube) defined by the surrounding 2^Q points at which data are defined. More generally, it is only necessary that points be defined in the space such that they have the topology of a partial grid: "holes" in the grid can then

* Durham Music Technology is a Collaboration between the School of Engineering & Applied Science and the Department of Music at the University of Durham

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be filled straightforwardly by interpolation, and the points are indexed by their position in
the grid, irrespective of their geometric position (Figure 1).

\[ \begin{align*}
(0, 0) & \leftrightarrow (3, 0) \\
(0, 1) & \leftrightarrow (2, 0) \\
(0, 2) & \leftrightarrow (2, 1) \\
(0, 3) & \leftrightarrow (3, 2) \\
(1, 0) & \leftrightarrow (2, 2) \\
(1, 1) & \leftrightarrow (3, 3) \\
(1, 2) & \leftrightarrow (1, 3)
\end{align*} \]

*Figure 1. A topological grid of points in \( \mathbb{R}^2 \)-space, with indices*

During a performance, \( N \) articulation parameters are measured, and these define an
articulation vector which selects a point in the hyperspace. In general, this point will not
be a point at which the control parameters have been stored, so some form of interpolation
is necessary. Simply selecting the nearest defined point can give rise to abrupt changes
in timbre and low sound quality [Bowler 1985], so some kind of interpolation is needed.
Temporal interpolation [Sasaki and Smith 1985] can produce undesired results when the
time taken for the articulation vector to move between defined points differs greatly from
the time constant of the temporal interpolator [Bowler 1986]. Other interpolation schemes
have had limitations such as being restricted to one dimension [eg. Grey 1975], or requiring
a complex triangulation of the space [eg. Bentley 1985]. The interpolation scheme suggested
by Schindler [Schindler 1984] is flawed and will not ensure first-order continuity for \( N > 1 \).

**THE INTERPOLATION SCHEME**

I have developed an interpolation method which ensures first-order continuity of the
interpolated functions, yet which requires only \( M \cdot N \) multiplies, compared with the \( M \cdot 2^N \)
multiplies required in more conventional schemes. The method partitions the measure
polytope into a set of simplex polytopes (the generic term for the sequence line, triangle,
tetrahedron .......), each having \( N + 1 \) vertices [Conner 1948]. If the set of points \( Q \) are
indexed by integers corresponding to their position in the grid, then, given a point \( P \)
contained within a particular measure polytope, the following algorithm will find a unique
simplex polytope containing \( P \) whose vertices are a subset of those of the measure polytope:

1. For each vertex in the measure polytope, Let \( C_i \) := the number of odd indices Modulo \( N + 1 \)
2. For each value of \( j \) from 0 to \( N \), select the point in the measure polytope with \( C_j := j \) which is closest to point \( P \).

The simplex polytope is defined by the \( N + 1 \) vertices thus selected.

Eight adjacent 3-Dimensional measure polytopes are shown in Figure 2, with their vertices
labelled with the number of odd indices modulo \( N + 1 \). It can be seen how the above
Algorithm selects points which form a tetrahedron. The complete set of tetrahedra in each cube fills it, and they have the common edge 01. Similar results are found for other values of $N$.

Figure 2. Eight adjacent measure polytopes in $S$-Space, labelled with the number of odd indices modulo 4 at each vertex.

One of the selected vertices, $V_0$, is chosen as an origin for a new set of basis vectors defined by the other $N$ vertices. The interpolating function is a hyperplane passing through each of the $N+1$ vertices, so the interpolated point $P$ must satisfy

$$P = V_0 + \sum_{i=1}^{N+1} \alpha_i V_i$$

(2)

The $\alpha_i$ are a set of scalar weights. If vertex $V_i$ has co-ordinates $x_{i0}...x_{iN}$, then (2) can be rewritten as $N+1$ linear equations of the form

$$x_{pj} = x_{i0} + \sum_{k=1}^{N} \alpha_k x_{kj}$$

(3)

The first $N$ of these may be solved for the $\alpha_i$. Using Gaussian elimination, this requires $O(N^3)$ operations [Sedgewick 1983], which is usually negligible for the problems of interest. Where this is not the case, the $\alpha_i$ can be pre-computed for a sub-grid of points: the point in the sub-grid nearest to the articulation vector is selected, whereas are defined weights for the relevant points in the surrounding measure polytope. This eliminates the additional computation at the expense of roughly doubling the storage requirements.

We are now left with an equation for the value at the point to be interpolated in terms of the values at the points in the simplex and the $\alpha_i$

$$y = y_0 + \sum_{i=1}^{N} \alpha_i y_i$$

(4)

This equation requires $N$ multiplies and is applied once for each of the $M$ synthesiser control parameters.
IMPLEMENTATION AND EXAMPLES

The technique is being implemented on a network of TMS0 floating-point transputers, and will be used to control a real-time additive synthesiser having 42 control parameters ($M = 42$) [Bowler 1989]. We anticipate that the initial implementation will allow for around six articulation parameters to be employed for each voice in real-time. Some of the computational advantages of the present scheme can be appreciated when it is realized that without interpolation around 10^28 words of memory would be required, while interpolations based on all the points in a hypercube would require 2568 multiplexers in real-time.

One possible application of the technique is the emulation of other synthesis techniques using this scheme with the additive synthesiser. If, for example, a small value of an articulation parameter causes the additive synthesiser to produce a dull tone, while a large value produces a bright tone, then the overall system is behaving as if a filter is being swept, as in subtractive synthesis. Similarly, articulation parameters can be mapped so as to emulate the modulation index in FM, or amplitude in wave shaping etc. Another possibility is to base the parameters stored in the space on the analysis of acoustic instruments: a two-dimensional space would be capable of accurately modelling the dependence of piano timbre on touch and pitch, for example. Possibly the most exciting aspect of the technique, however, is that it allows the specification of arbitrary performance instruments: It will be possible to design instruments without being fettered by the limits of particular synthesis algorithms or by their computational demands.

REFERENCES


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