MUSIC-TIME and CLOCK-PERIOD SIMILARITIES UNDER TEMPO CHANGE

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1. INTRODUCTION

At the last meeting of this group, we presented a general method for a set of subroutines for relating "music-time" or "last" and "clock-time" or "past" scores(*). This paper will continue that investigation, emphasizing certain characteristics of the score-time scores which are left unchanged under the operations of linear and equal-ripple tempo modifications. We will use the terms and models developed in our earlier paper. This we will speak of:

1. beats as a property of music-time scores (i.e., a half note gets two beats);
2. duration in seconds as a property of clock-time scores (i.e., a half note lasts 0.5 seconds);
3. the clock factor of the beat point as the period of the tempo at that beat point. Thus the clock factor in the time, in seconds, that one beat would have at the given tempo. The clock factor associated with a tempo of 60 is 1; with 120, 0.5; with 600, 0.1; and
4. the duration of some passage in seconds as the integral of the clock factor curve of the passage with respect to beats.

A complete set of formulas for computing tempo and duration using several scores curves is given below. Note that the second formula of each group is for duration and involves the integration of the clock-factor curve associated with the first formula of each group.

Sections 1-8 and the Appendixes of this article were written by John Powers in consultation with Philip Hatton's, whose structural ideas provided one of the key motivations for this effort, and John Rechaton, whose mathematical expertise provided the essential framework for our work. We are also indebted to Robert Carver of the Research Department of the University of New Hampshire for general help and advice in the preparation of these sections, particularly Appendices 2 and 11. Section 9 was written by Philip Hatton, but makes use of concepts and programming techniques developed by the other two authors. We refer readers who are interested in further study of Fort. Hatton's compositional
techniques to his doctoral dissertation (**2, **3).

1. **LINEAR TEMPO** (hyperbolic clock factor)
   1. T(bp) = (T1 x T2) / T1
   2. T(bp) = (T1 x T2) / (T1 / T2)

2. **EQUAL-VARIOUS TEMPO** and **CLOCK FACTOR**
   1. T(bp) = (T1 x T2) / (T1 / T2)
   2. T(bp) = (T1 x T2) / (T1 / T2)

3. **INVERSE EQUAL-VARIOUS TEMPO**
   1. T(bp) = (T1 x T2) / (T1 / T2)
   2. T(bp) = (T1 x T2) / (T1 / T2)

4. **HYPERCIC TEMPO** (linear clock factor)
   1. T(bp) = (T1 x T2) / (T1 / T2)
   2. T(bp) = (T1 x T2) / (T1 / T2)

where
1. bp is some best point.
2. T(bp) is the tempo at some best point.
3. T(bp) is the duration in seconds at some best point.
4. T1 is the initial tempo.
5. T2 is the final tempo.
6. N is the total number of beats in the passage.
7. bp is T1 / T2.
8. ln is the log, base e.

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2. FIVE CRITERIA FOR RHYTHMIC SIMILARITY

When the tempo of a passage is absolutely steady, the ratio of the clock times of successive beats is 1:1 and the ratio of the actual durations of the notes in the clock-time score are the same as the ratios of the durations of the notes in the music-time score. If a continuous, monotonic curve of tempo change is applied, the clock times of successive beats change continuously in length and their ratio is obviously, no longer 1:1. Clearly, the durations and the ratio of the durations of the notes in the clock-time score will no longer match those in the music-time score. There are five criteria we have found to be helpful when evaluating the similarity of passages undergone tempo change to passages played at a steady tempo.

1. The Shape of Tempo Change vs the "Straight Line"

Consider the start and clock times of successive beats in relation to the starting tempo (T1) and its associated clock factor (CF1) and the ending tempo (T2) and its associated clock factor (CF2). A curve which diverges slowly from T1 produces a realization which diverges slowly from a steady T1 realization of the notated passage. A curve which moves rapidly towards the goal tempo produces a realization which moves rapidly towards a steady, T2 realization of the notated passage. Perhaps the simplest criteria for evaluating tempo change is to compare the curve the tempo change function produces to the straight lines which would be produced by steady tempo realizations at either of the two specified times.

2. The "equal-to", "greater-than", and "less-than" Relationships

A second criteria has to do with the relationships equal-to (EQ), greater-than (GT), and less-than (LT). All tempo change functions destroy the EQ, relationship; notes of equal written value are not of equal clock-time value. Their clock-time duration depends on the tempo-change function being used and their place in the function. Some tempo change functions preserve the GT-LT relationship and some do not. Consider a case of a music-time rhythm of a dotted eighth followed by a sixteenth. The dotted eighth has a greater beat value than does the sixteenth. Suppose the tempo is slowing. If the result is great enough — if
the relationship between the beginning tempo and the end tempo to large enough — then the clock time durations produced will not have the same kind of relationship as the beat Durations. For example, we could choose a gizard that would make the clock time duration associated with the sixteenth note 80 or 1.8, the clock time associated with the dotted eight. These anomalous notations come from (1) the "direction" of dotted rhythm contradicts the "direction" of tempo change — in our case the direction of dotted rhythm was long to short while the direction of tempo change was short to long or (2) the difference in the given tempi is large in relationship to the change in dotted values. If our music-time rhythm had been a whole note followed by a sixteenth note, it would have taken a much more extreme tempo curve to cause the sixteenth note to take the longest time.

Tempo change functions which preserve the 0.711 relationship are intuitively simpler and of more general use. If a significant number of the directional relationships of the music-time score were found to be reversed in the clock-time score, serious mentions would be raised about the appropriateness of the tempo function being applied and the relationship of the performed piece to the notated one. In our case, however, we hope to note here that neither the equal-ratio nor the linear function preserves the 0.711 relationship for all cases.

3. The Maintenance of Rhythmic Proportionality

A third criteria concerns the proportionality of rhythms. Since, in a steady tempo, the ratios of successive beat durations is 1:1, all steady tempo performances of a passage "sound the same," in a sense. In its most obvious sense, proportionality of notated values is lost under tempo change; the relationship of a dotted eighth to a sixteenth is different depending on where they are located. The ratio of a dotted eighth followed immediately by a sixteenth is different from a dotted eighth to a sixteenth two beats later. More specifically, however, a sense in which a certain "motivic" aspect of proportionality and metric relations is preserved. The proportionality of a dotted eighth followed by sixteenth would no longer be 3/5 and the 0.711 relationship would not necessarily be preserved, but we might expect that whenever this "motivic" is found on the curve, it would be the same proportion. Further, the relationship of a dotted eighth to a sixteenth two beats later would set be 3/5,
would not be the case as a dotted eighth followed immediately by a sixteenth, and would not necessarily preserve the same relationship, but it is possible to take it the other way. The relationship between the two dotted eighths depends on the relationship between the two sixteenths.

The equal-ratio curve is the only tempo function that preserves this essential aspect of proportionality, and that by the same token one might hope to use it as the base function. Other functions produce different proportions depending on where the phrase is located in the curve. Thus the rhythmic motive of a dotted eighth followed by a sixteenth would produce differing clock ratios depending on its location in a musical passage with non-equal-ratio curves. In this case, non-equal-ratio curves produce passages which "sound different" depending on their tempo.

There is a second aspect of rhythmic proportionality which must be mentioned. Suppose we have a steady tempo, two voices both having the same rhythmic proportion, one twice as fast as the other. There is no continuous function that guarantees rhythmic proportionality of these voices under tempo change. Some specification of this problem will be discussed later in connection with multipoint cassettes.

4. Constant Duration and Tempo Change

A fourth kind of judgment about rhythmic similarity has to do with passages whose number of beats and tempi are related by some constant. Suppose we have a 16-beat passage at tempo 60 followed by a 20-beat passage at tempo 80. The number of beats is doubled but so is the tempo. Each passage will take the same amount of clock time. In general, if the number of beats in the passage and its tempo are both multiplied by the same constant, the clock time duration of each passage is the same. We now add the condition of tempo change.

Suppose the tempo accelerates from 60 to 120 over the first 10 beats and 120 to 240 over the next 20. Many functions may be multiplied so that they produce these equal tempi and so that the two passages take the same amount of clock time, but only one continuous function satisfies both these conditions - the linear function. The score of a linear function is 6 (6*10=60) when we move from 60 to 120; it remains 6 (6*20=120) when we move from 120 to 240. A discontinuous equal-ratio function, for example, would

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be needed to produce these tempo and duration changes since the equal-ratio function that moves from 60 to 120 over 10 bars would move from 120 to 240 over the next 10 bars, yet the next 20.

The "constant duration" property is important in many musical contexts since it allows one to treat proportional tempo change versus perceptual note-value change in an intuitively simple manner. The property will be of the first importance with reference to the rhythmic extraction procedures used in Phillip Ketner's music.

5. Second Order Similarities

Closely related to the properties discussed in 3 and 4 above is the in which a "new tempo" emerges from manipulation of an old. In such cases, a new "music-time" score seems to provide a simpler correlation with the performance than the real-time score which generated the performance.

Suppose a series of approximately equal durations were to result from a gradual acceleration of a music-time passage which contained gradually increasing note durations. Other things being equal, a listener would probably interpret the passage in an approximately steady tempo. Similarly, if a series of durations were to emerge which could be interpreted, even approximately, as one steady tempo, a listener would most likely make that interpretation.

If a pattern of tempo change is repeated over a set number of beats, this results in, at the very least, a pattern of "beat-duration" repetition. This pattern, even if somewhat irregular, may well be heard as "one tempo". This aspect unusual view of tempo is discussed in detail in connection with musical examples in Sections 8 and 9 of this paper.
1. **EQUAL-RATIO TEMPO CHANGES**

In an equal-ratio accelerando or retard, tempo change is proportionally the same from beat to beat. Consider an equal-ratio accelerando from T1 to T2. As tempo along the equal-ratio curve approaches T2, the absolute differences in tempo are smaller and the clock times associated with successive beats approach the clock time associated with the steady tempo, T1.

In general, the larger the T2/T1 ratio, over a set number of beats, the more the function's shape resembles a parabola; the smaller the T2/T1 ratio, the more the function's shape resembles a linear accelerando. The following table and the graphs included in Example 2 help make this relationship clear.

<table>
<thead>
<tr>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>(linear shape)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.318</td>
<td>2.5</td>
<td>(equal-ratio begin)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.5</td>
<td>4.65</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>10.5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>100.5</td>
<td>(1000)</td>
</tr>
</tbody>
</table>

The greater the T2/T1 ratio, the less time the accelerando takes and, in that sense, the faster it is. But the greater the T2/T1 ratio, the greater the proportion of the function that starts near T1. In that sense, the greater the ratio, the more closely we approach T2.

As the T2/T1 ratio is decreased, we approach the arithmetic mean of the two terms. To generate functional shapes which will approach T2 more rapidly than the linear function, one can use the inverse equal-ratio function or any of a large number of other functions described in our earlier article. (***)

In our particular implementation of the equal-ratio curve, we treat tetrads as the unit of accelerandi; thus, if a retard, the greater the T2/T1 ratio, the greater the proportion of the function that is near to T1 and, in this sense, the more rapidly it approaches T2.

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4. LINEAR TEMPO CHANGE

In a linear tempo change, tempi succeed each other by equal differences. An important negative feature of linear tempo change is that all rhythmic proportionality is lost, even that which we earlier called “motivic”. In this sense, linear transformations are not intuitively similar to steady tempi since the changes they produce are proportionally unique to each particle of the function. As we observed earlier, perhaps the most important invariant under linear tempo transformations is the preservation of clock time durations of beat segments whose tempi and beat lengths are related by some constant.

Linear accelerando and ritard are automatically mirror images of each other, thus no convention need be adopted here as was necessary in equal-rations. Let us consider, then, a linear tempo transformatics between T₁, the slower tempo, and T₂, the faster one. The duration or clock-factor curve associated with this tempo curve would be a hyperbolic decrease from (60/T₁) to (60/T₂). Suppose we compare these curves with an equal-rations accelerating from T₁ to T₂. We see that the tempo change is more rapid than equal-rations at the beginning of the curve and less rapid at the end. This means that, for any two given tempi, a linear accelerando approaches T₂ initially more rapidly than does an equal-rations accelerating. Conversely, a linear ritard approaches the slower tempo initially less rapidly than does an equal-rations function. Since the proportion of tempi change is continuously decreasing in a linear accelerando, there is a sense in which a linear accelerando resembles a steady tempo more and more closely. Conversely, in a linear ritard, the proportion of tempi change is continuously increasing and we diverge more and more from a steady tempo. Rapidly changing linear ritards can result in surprising differences in rhythmic proportions between music-time and clock-time sources.
5. LIBERAR AND EQUILIBRATED REALIZATIONS OF A SIMPLE PASSAGE

Consider the following written musical example:

**MUSICAL EXAMPLE 1**

![Musical notation image]

We see a passage that ritards as it moves up a D-major scale and accelerates as it moves back down. The metrical and rhythmic active is iterated at the beginning and end of each ritard and acceleration. The active employs a dotted eighth, sixteenth, and an eighth. The ratios of these in beats are 1.51.51. We will now examine four realizations of this passage. We alternately hear equi-ratio and linear functions applied. The first pair use equal tempi of 120-40-120. The second pair use equal tempi of 380-60-380. The following tables compare the ratios of durations produced by each realization. The graphs of these tempo functions may be seen in Example D.

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Philip Bates's music makes use of a method of pitch-class organization in which each particular pitch-class transposition of a pitch-class set is associated with a rhythmic transposition of a set of rest points. Pitch-class transposition is defined in the usual manner as the mod 12 addition of a constant, k, to the original set of pitch-classes, while rest-point transposition is defined as the multiplicities of the original rest-point numbers by a constant, l. This means that all intervals between rest-points in the transposed set are l-times as great as the original intervals. Later we will examine in detail the relationships between pitch transposition and rhythmic transposition in this system. We will now consider characteristics of the rest point itself. We generate rest points for our set from the formula (see §§2, pages 65-71):

\[(b^*\text{mod} z) = b \cdot n \cdot z\]

where

- b is an integer base
- n is an exponent ranging from 0 to c
- z is an integer modulus, where c is the cardinality of the set

We see an example based on powers of 3, mod 35

\[l = 1, 3 = 35\]

This modulus has been chosen to produce 12 rest points (c=12).

**Powers of 3, mod 35**

\[1, 3, 9, 27, 11, 33, 29, 17, 16, 13, 4, 12\]

(Best points in generated order, mod 35.)

Best points are generated in the order of powers of 3, mod 35 and all powers of 3, mod 35, are generated as best points. This multiplying the best point numbers by any power of 3, mod 35, will generate the order of the numbers and will generate all best points. It is a general rule that if some ordered set of numbers is multiplied by a constant, then the intervals between the numbers in that set are multiplied by the same constant. If we multiply our best points in generated order by any power of 3, mod 35, we
FITARD

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{func} & T2/T1 & R1 & R2 & R4 & R5 & R6 \\
\hline
at & 1/1 & 1.5 & .5 & 1 & 1.5 & .5 & 1 \\
& 2/1 & 1.55 & .46 & 1 & 1.35 & .48 & 1 \\
& 1/2 & 1.38 & .48 & 1 & 1.32 & .47 & 1 \\
& 8/1 & 1.11 & .44 & 1 & 1.11 & .44 & 1 \\
& 8/1 & 1.29 & .47 & 1 & .83 & .37 & 1 \\
\hline
\end{array}
\]

ACCELERANDO

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{func} & T2/T1 & R7 & R8 & R9 & R10 & R11 & R12 \\
\hline
at & 1/2 & 1.66 & .522 & 1 & 1.66 & .522 & 1 \\
& 1/2 & 1.503 & .527 & 1 & 1.62 & .514 & 1 \\
& 1/8 & 2.035 & .569 & 1 & 2.035 & .569 & 1 \\
& 1/8 & 2.636 & .606 & 1 & 1.73 & .525 & 1 \\
\hline
\end{array}
\]

In the ritardandi, the second and third notes of the rhythmic motive become relatively longer in relation to their music-time ratios. Thus the ratio of the first note to the last becomes smaller. Since a ritardandi contradicts the directionality of the written rhythm (long to short), it is possible that the ritardandi may modify the sense of the rhythmic relationships. This actually happens at the end of the linear ritarandi where the dotted eighth's value is shorter than the eighth by the ratio of .83.

In the accelerandi, the second and third notes become relatively shorter in relation to their music-time ratios. Although there is the possibility of a decelerational inversion between the second and third notes, this does not occur in our examples.

The equal-ratios function produces the same ratios for this motive within any one equal-ratio accelerando or any one equal-ratio ritardando, if that ratio is, of course, different for different T2/T1 ratios and are different depending on whether the passage is undergoing accelerando or ritardando. The linear function produces differing ratios within any application of the function.

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will simply rotate the set of beat points since they are in increasing powers of 3 already. The intervals between
the beat points also undergo a simple rotation.

Multiplication of the generated set

\[
\begin{align*}
1 & 3 & 9 & 27 & 11 & 33 & 25 & 15 & 16 & 13 & 30 & 12 \\
2 & 6 & 18 & 34 & 162 & & & & & & & \\
3 & 9 & 27 & 11 & 33 & 25 & 15 & 16 & 13 & 30 & 12 \\
6 & 18 & 54 & 108 & & & & & & & & &
\end{align*}
\]

(generated beat points) (Intervals not reduced
and 3) (Intervals, mod 3)

Suppose we now sort our attack points into numeric order. This forces them all within the span of the modulus,
which may be thought of as "one measure". The sum of the intervals will of necessity be equal to the modulus.

Sorted Beat Points and Intervals

\[
\begin{align*}
1 & 3 & 5 & 11 & 12 & 13 & 16 & 17 & 18 & 20 & 33 \\
2 & 1 & 5 & 2 & 1 & 1 & 3 & 1 & 10 & 2 & 4 & (3) \\
3 & 9 & 27 & 11 & 33 & 25 & 15 & 16 & 13 & 30 & 12 & (9) \\
6 & 18 & 54 & 108 & & & & & & & & &
\end{align*}
\]

(beat points) (Intervals, mod 3) (sum of intervals)

If we multiply the sorted beat points by 3, where 3 is
any power of 3, we produce intervals that are 3-multiples
of the original sequence. For example, if we multiply the
sorted order by 3, we produce intervals that are 3 times
the original. Further, the sum of the intervals is a
3-multiple of the original modulus. Thus if we multiply
the beat points of the sorted set by 3, we multiply the sum of
the intervals by 3 as well.

Multiplication of Beat Points and Intervals

\[
\begin{align*}
1 & 9 & 27 & 81 & 1 & 12 & 16 & 11 & 29 \\
2 & 3 & 15 & 6 & 3 & 3 & 9 & 2 & 36 & 37 & (9) \\
3 & 27 & 81 & 1 & 12 & 16 & 11 & 29 & & & &
\end{align*}
\]

(beat points * 3) (Intervals * 3) (sum of intervals)
If one original set size is measured, then each multiple of the original set will span 12n measures. The "closest" multiple to the original set occurs when n = 1, the number originally raised to a power. With reference to our earlier example the closest multiple is 3. In a sense there is no upper limit to this procedure. If we accept the module as a limit for multiplication, then the farthest remove multiple for our example would be 3×12 = 36, or 12n = 3. This number would multiply the original durations by 3, i.e., 1 1/2, 3 1/2, and would take 36 times as long to play as the original.

Since our rhythmic set is of size 12, we may associate with its elements a pitch-class set of size 12. There are optimal ways of making this association. Consider the rhythmic set in its generated order. Let us associate with this set a set of pitch-classes whose order is determined by some complete pitch-class cycle of the twelve-tone system. Complete pitch-class cycles are formed by those pitch-class-intervals which are relatively prime to 12, namely 1, 5, 7, and 11. We show an example based on the chromatic scale, the cycle generated by pitch-class-interval 1.

**Beat-point and pitch-class Association**

| 1 3 9 27 11 33 29 17 16 13 12 | generated beat points |
| 0 1 2 3 4 5 6 7 8 9 10 11 | generated pitch classes |

| 1 3 4 9 11 12 13 16 17 27 29 33 | sorted beat points |
| 0 1 6 2 4 11 9 9 7 3 6 5 | sorted pitch classes |

These pc numbers may be thought of as "order numbers" of the generated beat-point set. The shuffled order shows the order positions in the original generated set and the sorted set. Suppose scale constant, 1, is added, and 12, to the set of order numbers. This new set of order numbers shows the positions in the generated beat-point set of a second beat-point set produced by multiplying the sorted beat-point set by 12, where b is the longer base and n is the exponent between 1 and 11. Since we think of transpositions as the addition, and 12, of some constant, b, to a set of pitch-class numbers, we have here associated each possible pitch-class transposition of a pitch-class set with a multiplication of a rhythmic set.
In order to see why this is so, it is helpful to consider the following two strings of a base number raised to a power:

**Addition of a Constant to a String of Exponents**

\[ \begin{align*}
3^{0} & 3^{1} 3^{2} 3^{3} = 1, 9, 27, 81; \text{ intervals } 26,-24 \\
3^{1} & 3^{2} 3^{3} 3^{4} = 3, 9, 27, 81; \text{ intervals } 78,-72
\end{align*} \]

We see that by adding 1 to our string of exponents we multiply all numbers and intervals by 3. For the general case, adding a constant, \( n \), to all exponents in a string will multiply all numbers and intervals by \( b^n \), where \( b \) is our integer base. The numbers and intervals between numbers so generated may be reduced and \( n \). For most of our applications, the reduction and \( n \) will not be useful. This reduction would destroy the multiplicative relationship between our various simultaneous rhythmic statements. Finally consider our original pitch class structure as exponents for the integer base, \( b \), where \( b = 3 \).

**It’s as Two-point Exponents**

\[
\begin{array}{cccccccccccccccc}
1 & 3 & 4 & 9 & 11 & 12 & 13 & 16 & 17 & 27 & 29 & 33 & (values) \\
0 & 1 & 10 & 7 & 4 & 11 & 5 & 8 & 7 & 3 & 6 & 5 & (exponents)
\end{array}
\]

Adding 1 to all the exponents -\( s \), of necessity, multiply all two-point differences, that is all locations, by \( b \), the base. For the general case, adding some constant, \( n \), to all the pitch class numbers—thereby producing a pitch class transposition of the original pitch class set---will produce a rhythmic statement whose rhythmic intervals are \( b^n \) times as great as the original sorted rhythmic statement. This also means that one statement of the original set will take up the same amount of music-time as \( b \) statements of the original set at a pitch-class transposition of \( n \).
Exercises 1: Triple Transposition values and Parallels

<table>
<thead>
<tr>
<th>Voice</th>
<th>3 9 12 27 33 1 8 11 16 17 29 (values)</th>
<th>3 9 12 27 33 1 8 11 16 17 29 (values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voice</td>
<td>1 2 11 1 5 8 10 9 8 4 7 6 (exponents)</td>
<td>1 2 11 1 5 8 10 9 8 4 7 6 (exponents)</td>
</tr>
<tr>
<td></td>
<td>(Case)</td>
<td>(Case)</td>
</tr>
</tbody>
</table>

The following example shows two vertically aligned statements of our pitch/chroma data. The relative pitch transposition values are 0 and 1 and thus the relative tempo relations are 1/1. The original set moves at three times the tempo of the transposed set and thus each transposed set takes up three times as much music-time as three untransposed data. Voice 1 makes these statements of the untransposed set while voice 2 makes one statement at t=1. The beat-point intervals (bp's) -- the durations -- of the two voices are given as the second list of each voice. The third line shows the pitch classes (pc's) of each voice.

**Two Simultaneous Statements 1: 1/3, 1/2**

<table>
<thead>
<tr>
<th>Voice 1</th>
<th>1 3 4 9 11 12 13 16 17 29 33 (bp's)</th>
<th>1 3 4 9 11 12 13 16 17 29 33 (bp's)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 1 6 2 1 1 3 1 10 2 4 3 (bp's)</td>
<td>2 1 6 2 1 1 3 1 10 2 4 3 (bp's)</td>
</tr>
<tr>
<td></td>
<td>0 1 10 2 4 11 9 8 7 3 6 5 (pc's)</td>
<td>0 1 10 2 4 11 9 8 7 3 6 5 (pc's)</td>
</tr>
<tr>
<td>Voice 2</td>
<td>3 9 12 27 33 (bp's)</td>
<td>3 9 12 27 33 (bp's)</td>
</tr>
<tr>
<td></td>
<td>6 3 15 6 3 (bp's)</td>
<td>6 3 15 6 3 (bp's)</td>
</tr>
<tr>
<td></td>
<td>1 2 11 3 7 (pc's)</td>
<td>1 2 11 3 7 (pc's)</td>
</tr>
</tbody>
</table>

**********

<table>
<thead>
<tr>
<th>Voice 4</th>
<th>1 3 4 9 11 12 13 16 17 29 33 (bp's)</th>
<th>1 3 4 9 11 12 13 16 17 29 33 (bp's)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 1 5 2 1 1 3 1 10 2 4 3 (bp's)</td>
<td>2 1 5 2 1 1 3 1 10 2 4 3 (bp's)</td>
</tr>
<tr>
<td></td>
<td>0 1 10 2 4 11 9 8 7 3 6 5 (pc's)</td>
<td>0 1 10 2 4 11 9 8 7 3 6 5 (pc's)</td>
</tr>
<tr>
<td>Voice 5</td>
<td>1 9 13 14 (bp's)</td>
<td>1 9 13 14 (bp's)</td>
</tr>
<tr>
<td></td>
<td>3 9 3 30 (bp's)</td>
<td>3 9 3 30 (bp's)</td>
</tr>
<tr>
<td></td>
<td>0 10 9 8 (bp's)</td>
<td>0 10 9 8 (bp's)</td>
</tr>
</tbody>
</table>

**********

<table>
<thead>
<tr>
<th>Voice 8</th>
<th>1 3 4 9 11 12 13 16 17 29 33 (bp's)</th>
<th>1 3 4 9 11 12 13 16 17 29 33 (bp's)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 1 5 2 1 1 3 1 10 2 4 3 (bp's)</td>
<td>2 1 5 2 1 1 3 1 10 2 4 3 (bp's)</td>
</tr>
<tr>
<td></td>
<td>0 1 10 2 4 11 9 8 7 3 6 5 (pc's)</td>
<td>0 1 10 2 4 11 9 8 7 3 6 5 (pc's)</td>
</tr>
<tr>
<td>Voice 9</td>
<td>17 29 (bp's)</td>
<td>17 29 (bp's)</td>
</tr>
<tr>
<td></td>
<td>6 12 (bp's)</td>
<td>6 12 (bp's)</td>
</tr>
<tr>
<td></td>
<td>4 7 6 (bp's)</td>
<td>4 7 6 (bp's)</td>
</tr>
</tbody>
</table>
We mentioned earlier that our pitch-class set could be formed from any cycle that is relatively prime to 12. In order to relate this idea with the "exponential" interpretation of the pc numbers, it is sufficient to introduce the idea that the set of pcs may be multiplied by any constant, a, where a is one of the numbers relatively prime to 12. For the general case a must be relatively close to 1, where ℓ is the cardinality of the set. The following shows one such example under pc multiplication by 5 and 12.

**Two Signatures**

```
Voice a: 1 3 4 9 10 12 14 16 17 18 20 24 25 30 (bp's)
   m = 0 5 2 10 6 7 8 9 4 11 3 6 1 (pcs)
Voice b: 3 9 12 27 33 (bp's)
   m = 6 9 3 15 6 3 (pcs)
```

```
Voice a: 1 3 4 9 11 12 13 14 16 17 20 24 25 33
   m = 0 5 2 10 6 7 9 4 11 3 6 1
Voice b: 1 4 13 16
   m = 1 9 3 30
   m = 1 2 4 4
```

```
Voice a: 1 3 4 9 11 13 14 16 17 20 24 25 33
   m = 0 5 2 10 6 7 8 9 4 11 3 6 1
Voice b: 11 17 25
   m = 6 12 (9)
   m = 6 11 6
```

We should now be able to understand more fully the close relationship between the pitch-class and key-point operations of this system. A pitch-class is, by definition, simply one part of an evolved scale exponent -- pc e needs a registral marker to become a frequency determinant. If we give it the registral 5, we produce the octave, 5th number, 0.06. This is as odd a form of exponent in the sense that...
.66 should be converted to .5(6/12) before the exponent is associated with its correct base, 2. Since we have just shown how pc mutes may be regarded as beat-point exponents, this means that the same numbers are serving as both rhythmic and pitch exponents.

Each statement of the basic set or case of its transpositions may be spoken of as a "set instance". It is often convenient to think of the "faster moving" sets of the above combinations as the "background". The "slower moving" sets are extracted from this background and may be spoken of as "extracted sequences". One straightforward interpretation of this kind of rhythmic scheme would have "slower and slower" extracted voices against a steady background. Another interpretation would allow each set instance to state the same duration against an appropriately adjusted, even faster moving, background. Both these interpretations involve different tempi and may be easily extended to involve changing tempi.

7. RHYTHMICALיגHY AND ТемПЧАНЕ

In an earlier discussion, we contented that tempi change destroys rhythmic proportionality in its simplest sense. In a steady tempo there are four quarts per whole note and the quarters proceed at four times the speed of the whole. Consider the following passage played at a steady tempo.

MUSIC EXAMPLE 2

\[ \text{Diagram of musical notation} \]
The following equations are true for steady tempo:

\[ \frac{b}{a} \rightarrow \frac{c}{a} \text{ since all quarter notes are equal.} \]
\[ \frac{b}{a} \rightarrow \frac{c}{b} \text{ since the ratio of all "equal note values" is } \frac{b}{c}. \]

In an accelerating, the durations of the quarter notes become progressively shorter, similarly, the second whole note is shorter than the first. Thus

\[ \frac{a}{x > 1}, \text{ where } x = b \cdot c \cdot d \cdot e \cdot f \cdot g \cdot h \]
\[ \frac{a}{x} > \frac{b}{c}. \]

If we consider an equal-ratio accelerator, then

\[ \frac{b}{a} \rightarrow \frac{c}{a} \rightarrow \frac{d}{b} \]
\[ \text{[e]}/[f] \rightarrow [g] \rightarrow [h] \rightarrow [i] \text{ does not equal } \frac{a}{b} \] but \[ \frac{[e]/[f]}{[g]} = \frac{[h]}{[i]} \text{ does not equal } \frac{a}{b} \]
\[ \text{the } \frac{a}{b} \text{ does not equal } \frac{[e]/[f]}{[g]} \] but \[ \frac{[e]}{[f]} \text{ does equal } \frac{a}{b} \times 1. \]

The above equations make clear that, in equal-ratio accelerando, the proportional rhythmic relations among simultaneous voices are lost. An "ideal" function to avoid this problem would be one in which proportionality is maintained over all possible comparisons. Thus

\[ \frac{[e]}{[f]} = \frac{[g]}{[h]} = \frac{[i]}{[j]} \]

Unfortunately, such a twofold-change function contains a logical contradiction and cannot exist; only the steady tempo function fulfills this condition.

These observations are generic to our basic example since the identification of rhythmic proportion in extended-type acceleration canons may be of perceptual importance. If we apply an accelerando to the earlier "achromatic", two-voice example, we can forecast that the proportional proportion of the faster voice will be longer than the same as those of the slower. Suppose an accelerando used an equal-ratio function and moved from 440 to 500 over the 35 beats of the slower voice. (See Section 6 and Appendix III, as well.) The following table summarizes the results of this accelerando by comparing the musical-time proportions with the actual resulting durations for the first five notes of each voice:

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A linear acceleration produces the following results:

<table>
<thead>
<tr>
<th>props</th>
<th>2</th>
<th>1</th>
<th>5</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>v 2 decs</td>
<td>1.127</td>
<td>.5374</td>
<td>2.4688</td>
<td>.87666</td>
<td>#622</td>
</tr>
<tr>
<td>v 2 ratios</td>
<td>2.695</td>
<td>1.261</td>
<td>5.855</td>
<td>2.0927</td>
<td>#1</td>
</tr>
<tr>
<td>v 1 decs</td>
<td>.29172</td>
<td>.19281</td>
<td>.93036</td>
<td>.36027</td>
<td>#7733</td>
</tr>
<tr>
<td>v 1 ratios</td>
<td>2.209</td>
<td>1.0773</td>
<td>5.2691</td>
<td>2.2377</td>
<td>#1</td>
</tr>
</tbody>
</table>

Neither of these curves, nor any other curve, allows each voice to present the same proportional relationship. This is an important limitation on the coherence of metrical canons undergoing tempo change. Indeed, it suggests that other rhythmic features may be more important compositional resources of metrical-type methods than the fact that the voices are, in some sense, of the same proportional rhythms.
8. SIMULTANEOUS LINEAR ACCELEBRATIONS

We will now construct a musical example based on the data sets listed on page 20 of Section 6. The example will have the following characteristics:

1. three voices will be employed,
2. each voice will enter at a pitch-class transposition of \+5 in relation to the immediately preceding voice,
3. the pitch-class levels of the entries will thus be \(0, 5, 10\),
4. each voice will make four statements of its pitch rhythm net,
5. the last three statements of each voice will take the same amount of time as the first statement of that voice,
6. the first statement of each voice will take the same amount of time,
7. each new voice will enter after the complete statement of the preceding voice,
8. each new voice will begin at the same time as the second statement of the voice already playing and will take up the same amount of time as three statements in that previous voice,
9. each voice will begin at the same time and accelerate to three times that tempo over its first statement,
10. each voice will accelerate to three times the original tempo (or three times the new) over its final three statements,
11. one continuous listen function will be used to control the shape of a crescendo for each voice,
12. voice 1 will begin in register 9 (or note 9,001);
voice 2 will begin in register 4 (or note 6,001);
voice 3 will begin in register 9 (or note 9,10).
13. registrations will be used to help create the voices distinct with the exception. The "slow motion", extracted voice will always be doubled at the unison rather than the octave, when such duplications arise between the "relatively background" voice and the extracted voice.

A taped example is played here. See Appendix XII for WAMS2 Printout of that example. See Music Example 5 for a "rotated version".

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we note that:

1. the use of the linear functions has brought about the rhythmic alignment of the voices through the use of the constant functions.
2. in order to make foreground statements (extracted sequences) take the same amount of time, it was necessary for the background to be continuously accelerating. In fact, the original "background voice" disappears with the entry of voice 3. If it had continued as a "real line", it would have eventually reached the tempo of 300 beats per minute.
3. the durations (actually attack point differences) of each "extracted voice" are the same as those of the original voice. For this fact, it is obvious that each "extracted voice" takes the same amount of time as the original and that each "har" takes the same amount of total clock time. The temporal relations of "one bar to another" are thus exactly the same, and, in a special sense, each entry is in the "same tempo". (See Section 3.)
4. the proportions revealed by any two simultaneous statements are not and cannot be the same.
5. the proportional differences in rhythmic value are less as the pieces accelerate since a linear acceleration approaches, in this sense, a straight line (see Section 4).

This example shows a three-voice extraction from a four-consonance statement of a set undergoing acceleration. It is shown twice, once using the straight-line curve and twice using the equal-ratios curve. (Min = the numerator beats/minute)

Example V

The Straight Line

Max = 300, accel. ... al, Max = 300, accel. ... al, Max = 400

The four background statements (times are in seconds):

day (time of final attack) 30.82

The three extracted voices:

3.38 [c1: 0.150 0.510 0.860] (Max = 300)

The Equal-Ratio Curve (This is an attempt to produce the same results as above.)

z761

(31 x 761) = 23406

[1.60] (31 x 761) = 23406

Notice that here the times do not line up at all. See example W for a detail.

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STRAIGHTLINE
A BACKGROUND SETS

EXAMPLE W

1. Note that the background differences here are minimal, turning from back to front.

2. Note that with the above, a third cosmic voice could be added, beginning with the C of E, which would reproduce exactly the background. With the equal motion version below this is impossible.

B EQUAL MOTION

BACKGROUND SETS

1. Note that the effect would be more here, as one of the voices is introduced at the beginning of the background sections, just above the nuclear voice, as if some clouds from the background.

2. The nuclei are not level, as if some clouds from the background.
9. TEMPO CHANGE IN STEADY-STATE [T. Bartstone]

The preceding musical example by Prof. Roger was the same pitch/time set as the one discussed in my Examples V and VI. From these examples and from the preceding musical one, we see that the use of the equalization curve preserves perceptual identity according to the tempo change, while extracted sequences that are meant to duplicate previous rhythmic relationships do not act and cannot. On the other hand, use of the straight-line function, while of necessity altering perceptual identity of successive statements, does so in order that extracted sequences can duplicate exactly the theories of set instances which occurred previously at the same tempo.

As we will see below, this fact is the only reason for my choice of the linear function and the associated hyperbolic clock factor as part of my compositional methods. Yet this fact allows me to think of my set being composed as presenting one particular primary tempo of my own choosing and not any one of a number of simultaneous temps.

Most of my compositional concern passes which contains from three to five voices, each undergoing tempo change. It is at the medium tempo that the most limitation occurs. If we take a situation where, for instance, the tempo change is from Tempo 1 (T) to twice that tempo (2T) over the statement of one set instance, and from that new tempo to twice again over the next two statements (leading at 4T), it is clear that the relative amount of tempo change over the third statement is less than that over the first. In short, the faster the general relative tempo, the slower the rate of change. Conversely, the slower the general tempo relatively, the greater the rate of change. The rate slower statement, the one which would have preceded statement number 1 on the same tempo function, would double its tempo over the latter half. In Examples II, I have shown on the left the relative tempo changes of various sets and on the other according to the general tempo and according to the curve just described. On the right I have gathered the together, not exactly adjacent time, but simply on a curve illustrating the differences in rate of change between and statements.

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Note that the radius tempi, those within the circle, are those among which there is the greatest variety in proportionality. Please keep in mind that for my purposes this variety is desirable as well as necessary to maintain tempo identity. One might say that this variety is the overriding reason for using tempo change when it is not perceived in the traditional way, e.g., as stringendo, rallentando, etc. In a passage to which the above tempo relationships apply, it is those statements which move from T to 2T which present what I call the "basic tempo" of the passage. I call this the basic tempo even though tempo change is involved, because the tempo change is not perceived as such. In fact, the choice of the degree of tempo change is often dictated by concepts of identity of rhythmic figures within set figures (and not just between them). This is the kind of redundancy which is characteristic of steady tempo and, as can be seen from Example 7, tempo change is necessary in order to create this identity. It is in this sense that I speak of "tempo change as steady state."
EXAMPLE Y

Steady Tempo

Background

First level extraction

Second level extraction

Here the overall durations of the two three-note groups are quite different. Note that the second group (4 3 2) is the retrograde of the first.

Accelerando

Here the overall duration of the same groups is the same.

Background Note that in the background "the difference persists."

First level extraction

Second level extraction

...
How the basic tact, however fast or slow it may be arbitrarily, can be altered in the usual way in the performance completely independently of the considerations discussed here. In fact, there is nothing in this discussion which predicts the application of an oppressiveizard or atemporsical here and there, even if it appears to contradict the tempo curve being used.

The distinction is between two different uses of tempo change. One is really on the performance level, while the other is entirely structural, having to do with harmonic example as well as metric identity. As shown in Example 8, the rhythmic relationship of the attack points of several simultaneous voices is the first and foremost reason for the preference for one kind of tempo curve over another.

To sum up, there are three advantages to the choice of the hyperbolic clock factor which make it my choice in most of my recent music. They are:

1. the presentation of harmonic content,
2. the identifiability of melodic material which recurs, and
3. the singularity of tempo at any given time.

The latter, of course, not only is achieved at the expense of isotonicity between statements at different tensi, but, in fact, depends on it.

Since what is true of combinations like that of Example 1 also holds true for combinations of combinations, there are a great many possibilities of combination, including those of disjunction the addition of voices to a given passage) as well as recombination (the removal of voices from a given passage). There are also, of course, other models than 30. the models of the above examples (for a larger musical example using the above set see **). Until recently, all the models I have used have been factors of 2**12-1 or of 3**12-1**2, 2**8. The exponent is 12 because the group of pitch classes is of order 12.

For several years, I have been using the computer to realize various studies which employ 53-tone equal temperament. Aside from the nearly perfect harmonic "fifths" and "thirds", which is well known, there is the fact that 53 is a prime number. This means that any interval within the group can generate the entire group.
Therefore the number of possible sets (excluding transpositions but not retrogrades or inversions) for the use of my techniques using powers of 2 is not \((2^{p} + 53 - 1)/13\), but \((2^{p} + 53 - 1)/15\), a larger number indeed. For convenience, I use as my modulo the number 631, one of the prime factors of 2^{p}53 - 1. (See Appendix II.)

Example 2 shows the disposition of the four voices of the taped example yet will hear. This is one of the four 'quartets' from the Sixth Suite of Nine Studies for Cicerone, which were completed in 1978. This example was realized on the DEC50 computer at the University of New Hampshire using MUSIC40P and a set of special EASSI tempo and note-generation routines.

(Taped example was played.)

While my pitch-rhythmic techniques do not necessitate computer realization, the complexities involved in such 11-tone studies, particularly when curves of tempo change are applied, make such composition without the computer almost insurmountably difficult.

A visual aid showing the disposition of the four voices in the recorded example. The relative tempo relationships are shown at left (S.T., S.3T, etc.)

Some of the starting and ending tempi are also shown.

A \((<S.T.>)\)

\[\text{Accel} \quad \text{From the 6th \% to the end}\]

B \((<S.T.>)\)

\[\text{Accel} \quad \text{From the 6th \% to the end}\]

T \((<S.T.>)\)

\[\text{Accel} \quad \text{From the 6th \% to the end}\]

\[\text{Accel} \quad \text{From the 6th \% to the end}\]

There are only three entire statements of the 53-tone set presented by individual voice. However, in other examples, the overall combination creates several 'inner-moving' background statements.

The fifth quartet from the last of "Nine Studies for Computer."
Barstow rhythmic sets are generated from the formula:

\[(b+m)*z \mod n\]

where

- \(b\) is the base number
- \(m\) is a multiplier
- \(z\) is the modulus and
- \(z\) ranges from 0 to \(c-1\) where
- \(c\) is the length of (number of elements in) the set.

We may make the following observations:

1. the largest modulus is \((b+c)-1\). We will call this particular modulus \(y\).
2. other and smaller \(z\)'s may be found providing:
   1. the particular modulus, \(z\), is larger than \(c\), the size of the set.
   2. the particular modulus, \(z\), is a factor of \(y\).
   3. in the formula \(y=(b+m)-1\), \(m\) is not a factor of \(c\).
   4. the new \(z\) is a factor of a larger \(\text{barstow}\) that fails because of criteria 3 above.

Thus for sets of size 12, base 2:

1. \(2*12-1 = 23\) is \(y\), the largest modulus.
2. the following factors of \(z\) will work as new \(z\)’s:
   - 12, 10, 8, 6, 4, 3, 1
   - 23, 21, 20, 19, 18

3. other factors of 235 will not produce enough beat points to work as new \(z\)’s since they violate one of the criteria listed above. For example:
   1. 63 will not work since \((2*64)-1 \equiv 63\) and 6 is a factor of 12.
   2. 15 will not work since \((2*16)-1 \equiv 15\) and 4 is a factor of 12.
   3. 21 will not work since 2 is a factor of 63 and 63 will not work.
We may make the following observations about the multipliers which may be used:

1. No multiplier can produce a set of larger cardinality than that of the original set.

2. If the multiplier is included in the original set, we simply preserve the original series and no new beat points are created.

3. If the multiplier is not in the original set and in relatively prime to the modulus, a new set of the same cardinality as the original will be produced.

4. If the multiplier is a factor of the modulus then the maximum cardinality of the set it will produce is $\frac{m}{\text{gcd}(m, \text{multiplier})}$, where $\text{gcd}(x, y) = x$ and $x \equiv 1 \pmod{\text{module/multiplier}}$ provided $\frac{m}{\text{gcd}}$ is no longer than the size of the original set. The actual cardinality of the set depends on the number of numbers in the original set which are distinct mod $\frac{m}{\text{gcd}}$, and thus the actual size of the set may be less than $\frac{m}{\text{gcd}}$.

5. If the multiplier is included in a set produced by one of the above operations, then multiplying the original set by the multiplier will produce a permutation of the set in which the multiplier is included.

6. Sets produced by multiplication are either totally identical in content or totally distinct, provided the base and modulus remain the same. They cannot share some numbers and not share others.

7. All possible numbers up to and including the modulus may be generated by multiplying the set by appropriate multipliers.

Two examples of this procedure follow:
TOWERS OF HANOI

If we try all possibilities for 0-15, and 15, we may make the following observations regarding the above example.

1. multiplying by any power of 2, and 15, yields a permutation of the numbers in the original set.

2. multiplying by 3 yields a new set of size 15 since 3 is relatively prime to 15 and is not included in the original set.

3. multiplying the original set by any number found in the "matrix" set results in a set that is a permutation of the "matrix" set.

4. multiplying by 5 can produce, at maximum, a new set of cardinality 6 since (15/3)-1 is 6. Since there are only 3 numbers distinct, and 7, in the original set, multiplying by 5 will result in a new set of cardinality 3.

5. multiplying by 7 can produce, at maximum, a set of cardinality 4 since (15/7)-1 is 4. Since there are four numbers in the original set which are distinct, and 7, a new set of cardinality 4 is produced by this operation.

6. If the original set is multiplied by 3, the "missing" three numbers which are distinct, and 7, are produced. Multiplying this set by 5 will produce the other numbers which could have been found in the "matrix" set. Multiplying by 3 and then by 5 is equivalent to multiplying the original set by 15.

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Powers of 2, mod 35

1 3 9 27 1 33 25 7 16 12 8 6 19 24 14 6 29 22
2 6 18 19 22 7 21 5 15 30 20 25 7 21 26 16 7 21 27 16
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

1. multiplication by any power of 2, mod 35, will simply permute the numbers in the original series.

2. multiplication by 2 will produce a new set of size 12 since 2 is relatively prime to 35 and is not in the original set.

3. multiplication by 5 will produce a new set of cardinality 6 since there are 6 numbers which are distinct, mod 5, in the original set.

4. multiplication by 7 will produce a new set of size 6 since there are 6 numbers which are distinct mod 5 in the original set.

Finally, consider a set based on powers of 2, mod 4095, the largest modulus. Suppose the resulting set were multiplied by 63, mod 4095. In order for there to be 12 elements in the resulting set, there must be 12 numbers distinct, mod 43, in the original set (4095/65=63). For there are only 6 numbers, and we produce a new and smaller set even though we are multiplying by a number which is itself a valid modulus. If we were to multiply by 63, however, we would produce a set of size 12 since there are 12 numbers which are distinct, mod 65, in the original set.
<table>
<thead>
<tr>
<th>Time</th>
<th>First Voice Duration</th>
<th>Second Voice Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>67. 0.533 1.678 9.00</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 1.661 0.560 8.05</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 2.161 2.116 8.12</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 4.357 0.764 9.12</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 1.121 0.362 9.08</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 5.083 0.350 9.07</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 5.263 5.865 8.09</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 5.263 0.350 9.04</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 6.257 2.671 8.11</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 6.800 0.462 8.33</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 10.261 0.866 8.04</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 11.128 0.408 9.07</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67. 11.734 0.385 9.02</td>
<td>(V 1, 2nd STATEMENT)</td>
</tr>
<tr>
<td>1</td>
<td>67. 12.719 1.100 6.05</td>
<td></td>
</tr>
</tbody>
</table>

The second voice repeats the original durations of the first voice with a slight adjustment of the second voice.

1. 68. 11.119 1.070 6.05 (V 2 RESTATE)
I 67. 76.517 1.169 9.00
I 67. 20.666 0.283 9.85
I 67. 20.769 0.407 9.80
I 67. 21.776 0.158 9.70
I 67. 21.375 0.076 9.60
I 68. 21.375 0.462 9.58
I 68. 21.413 0.078 8.07
I 67. 21.406 0.230 8.06
I 67. 21.721 0.076 8.04
I 67. 21.757 0.727 5.11
I 68. 21.747 0.486 5.11
I 67. 22.524 0.140 8.73
I 67. 22.663 0.400 6.01
I 68. 22.163 0.278 6.06
I 68. 22.597 0.172 6.01 "last note, x 1.5"
I 57. 21.261 0.385 6.00 "(1, 2, 2nd statement)"
I 67. 22.656 0.188 7.10

Third Voice: Express the duration of the first voice
while the second voice continues.

Note that this second passage is the same as the
second phrase passage.
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I 67, 31.305 2.407 7.07
I 67, 31.712 0.158 7.03
I 67, 31.970 0.178 9.01
I 68, 32.570 0.462 9.01
I 68, 32.549 0.176 9.01
I 67, 33.021 0.230 7.02
I 67, 33.256 0.776 8.09
I 67, 33.323 0.727 9.04
I 67, 33.333 0.866 5.06
I 67, 34.055 0.160 4.08
I 67, 34.150 1.279 6.11
I 68, 31.958 0.468 9.11
I 67, 34.672 0.174 6.06 (LAST NOTE, W 3)
I 67, 34.104 0.185 9.10 (W 3, WC STATEMENT)
I 67, 35.192 0.184 6.02

THIRD STROKE DURING TIMES AS SECOND VOICE

THIRD VOICE CONTINUES WITH ITS LAST THREE STATEMENTS:

I 67, 35.377 0.890 10.09
I 67, 36.268 1.336 7.08
I 67, 36.504 0.164 10.08
I 67, 36.768 0.162 7.09
I 67, 36.829 0.470 10.07
I 67, 37.450 0.152 10.02
I 67, 37.532 1.610 10.09
I 67, 38.963 0.241 9.01
I 67, 35.224 0.503 10.04
I 67, 35.727 0.283 8.11
I 67, 40.080 0.235 8.10
I 67, 42.324 0.115 8.03
I 67, 44.849 0.559 8.20
I 67, 46.599 0.215 8.08
I 67, 41.513 0.105 9.06
I 67, 41.370 0.105 9.05
I 67, 41.924 0.309 7.07
I 67, 41.733 0.105 8.02
I 67, 41.634 0.240 9.09
I 67, 42.794 0.182 5.01
I 67, 42.976 0.304 9.06
I 67, 43.330 0.173 9.11
I 67, 43.378 0.169 6.13
I 67, 43.757 0.243 10.02
I 67, 43.910 0.247 10.00
I 67, 44.547 0.158 5.08
I 67, 44.606 0.079 7.06
I 67, 44.888 0.078 9.05
I 67, 44.562 0.236 9.07
I 67, 44.732 0.076 9.02
I 67, 44.867 0.727 9.09
I 67, 45.756 0.100 6.31
I 67, 45.734 0.274 8.04
I 67, 46.788 0.134 8.14 (LAST NOTE, W 3)

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REFERENCES


3. continued in Perspectives of New Music, Fall-Winter 1972, pages 92-111.