MSC: a computer assisted system integrating music and video through magic squares as compositional models

Paul A. Oehlers* and Christopher H. Mich†

*Computer Science, Audio Technology, and Physics Department, American University
oehlers@american.edu
†Production Department, QVC Broadcasting
CHRIS_H_MICH@qvc.com

Abstract

Magic Square Composition is a collaborative computer assisted system that generates an output manifested as either film or live and electro-acoustic music that employs formalization to determine structure. The composer and filmmaker developed the system by deriving formal construction from a magic square, which consists of a series of numbers arranged so that the sum of each row, column, and diagonal are the same amount. The composer and filmmaker independently select routes, choosing identical routes in some sections and differing ones at other points, which are then assembled to form the structure of the piece. A description and implementation of the system is presented through the composition MFL.

1 Background

Completed in 1999, Magic Square Composition (MSC) was conceived as a computer assisted system that would enable independent expression by composer and filmmaker, generating different realizations of artworks using the same mathematic constraints. Our primary goal was to develop a system that provided the same structure for all visual and auditory components without dictating content. MSC was designed in the tradition of exquisite corpses in visual art, as no discussion regarding the selection of material is necessary once the structure is determined. The composer only sees the completed film once it is synchronized with the music. The filmmaker only hears the final version of the music once he had completed the film.

1.1 Background of magic squares

In order to examine the construction of MSC, it is necessary to understand the basic principles of magic squares. A magic square is a square matrix of $n$ rows and columns; the first $n^2$ integers are arranged so that the sum of each row, column and diagonal is the same amount. A five by five or order five square, for example, would contain the numbers 1 through 25, each occurring only once. Adding the numbers of each row, column, or diagonal would generate the same total, 75. An order six square, in comparison, would contain the numbers 1 through 36, and the sum of each of the rows, column, and diagonals would each equal 111. Magic squares can be classified into three types: odd, doubly even ($n$ divisible by four) and singly even ($n$ even, but not divisible by four). (Andrews 1960)

Before constructing our system, we examined other methods employed by other artists and composers and manifestations of their respective magic square artworks.

The compositional system of Sir Peter Maxwell Davies is typified in Ave Maria Stella, written in 1975. Davies uses the magic square of the Moon, one of the “Ptolemy Magic Squares” in De Occulta Philosophia, a book on magic by Heinrich Cornelius Agrippa von Nettesheim, written in 1531. (Agrippa 2002) As this is an order nine square, Davies reduced the square by modulo 9 to form a Latin square, a square of matrix $n$ rows and columns, whose cells contain $n$ different symbols so that no symbol occurs more than once in any row or column. He then correlated these numbers to the pitches of a plainsong with the same title and used the positions in the square to control duration. Although the piece contains nine movements, the use of the square is only to control these local aspects of the piece. (Warnaby 2006)

In the research presented on the compact disc recording Banjaxed, Zack Browning developed a system that employed routes through the magic square, but the individual positions within the square do not correspond to a specific theme. The number seventeen may appear twice with a different set of themes, density, and orchestration parameters. This system remains constant over a number of compositions written beginning in 1986. (Browning 2001) This documented history made this body of work ideal for study. Browning’s earlier experiments in the use of magic squares are also worth noting, particularly due to his departure from these compositional methods toward those employed in the compositions on Banjaxed. Browning attempted to integrate magic squares into every aspect of his music, but found the more appealing aspects of using the square was to create a means by which the change in the individual position was apparent. (Browning 2006)
artworks by Paul Klee and others were studied, but discussion of these is beyond the limitations of this paper.

1.2 Background of the second realization of MSC: MFL

The first realization of a composition was for experimental purposes. It resulted in a composition for orchestra, which revealed several imperfections in the preliminary composition system. These were corrected before the second run was started. MFL for piano, film, and electronics, was begun as the result of a commission by composer/pianist Mei-Fang Lin. The title comes from her initials.

1.3 Selecting the magic square for MFL

The particular magic square employed in the construction of MFL is the magic square of the Sun, one of the “Ptolemic Magic Squares” in De Occulta Philosophia. The magic square of the Sun is an order six, singly even square, as when divided through the center intersection of the square forms quarter squares that are of an odd order, or root. While most magic squares are associative (containing skewly related numbers opposite one another), this particular square is the only one of the Ptolemic squares that is not. (Agrippa 2002)

<table>
<thead>
<tr>
<th></th>
<th>6</th>
<th>22</th>
<th>3</th>
<th>24</th>
<th>35</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>11</td>
<td>27</td>
<td>28</td>
<td>8</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>14</td>
<td>16</td>
<td>15</td>
<td>23</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>20</td>
<td>22</td>
<td>21</td>
<td>17</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>29</td>
<td>10</td>
<td>9</td>
<td>26</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>5</td>
<td>33</td>
<td>4</td>
<td>2</td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. The magic square of the Sun

We selected this magic square due to its use other systems. The symbolism of using this square was indeed one factor, however, by analyzing the results of the earlier systems, particularly Browning’s, the results of these other systems made the selection of this particular square apparent.

2 Construction

As it was critical that the timing for the music and the film remain consistent, every global element of MSC is determined by the magic square. Routes through the magic square are mapped onto both the musical and visual structure. The unique location of each number within the square and its position within the route is paralleled in the musical score by a particular style, density, timbre, orchestration, amplitude (dynamics), and digital signal processing. The individual numbers in the square are used to represent the number of beats of that section.

In the film, individual positions determine the scene or scenes and the visual effect or filter. Although the specific process of correlating the scene or combination of scenes corresponds to individual position of the square is different, the principle is the same. The unifying element of all of the scenes is their inclusion of a type of human propulsion. Even in shots with no motion, such as the close up of a dancer’s feet, the focus is on this theme. In total there are twenty-four distinct scenes, six scenes filmed at each of the four different locations.

Los Angeles area
Actors walk up stairs
Bicycle
Plikhha Airport
Construction workers
Los Angeles Amtrak Station

West Chester, Pennsylvania
Actor at QVC Offices
Actor smoking
Chris Mich close up
Dancer’s feet close up
Film Camera
Flashlight sequence

Hollywood State Park, Pennsylvania
Olympic athlete (front)
Olympic athlete (back)
Running sequence
Sneakers close up
Trees

Figure 2. Twenty-four scenes in MFL

2.1 Constructing musical themes

The themes of MFL were designed to contain micro- and macro-cellular structure. This was accomplished through weighted probabilities, primarily Markov chains, for each series of possibilities. The six themes in MFL were each constructed by employing different compositional devices to ensure a unique identity and placed into specific frequency ranges to enable the pianist to simultaneously perform several of the themes simultaneously. These ranges of each theme remain a static parameter throughout the piece.

As MFL employs an order six magic square, six distinct musical themes were created. Each of these themes is of differing lengths, in order to ensure asynchronous repetitions against the others. The basic unit of measurement dictating tempo is beats per minute with quarter note equaling 144. Figure 3 describes each o the themes in regards to these musical parameters.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Duration (in beats)</th>
<th>Range (in pitch space)</th>
<th>Micro-cellular structure</th>
<th>Macro-cellular structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>28</td>
<td>D &amp; G</td>
<td>aabbb abbb abbb abbb</td>
<td>AABRA</td>
</tr>
<tr>
<td>B</td>
<td>29</td>
<td>A flat 4 - F flat 6</td>
<td>aabb cdde cdde cdde</td>
<td>A^A/B^B A^A</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>E flat 2 - A flat 5</td>
<td>abcd efgh efgh efgh</td>
<td>ABAB</td>
</tr>
<tr>
<td>D</td>
<td>24</td>
<td>G1 - D7</td>
<td>aabbb cde cde cde</td>
<td>ABAB</td>
</tr>
<tr>
<td>E</td>
<td>22</td>
<td>C1 - E2</td>
<td>abcd abcd abcd abcd</td>
<td>ABCD</td>
</tr>
<tr>
<td>F</td>
<td>30</td>
<td>D flat 6</td>
<td>abcd abcd gabc abcd gabc</td>
<td>ABCC/CD</td>
</tr>
</tbody>
</table>

Figure 3. Basic structure of the themes in MFL

The micro-cellular cells do not remain consistent in respective themes. The cell “a” in theme A is different from “a” in theme B. The cells are roughly all the same length from an eighth note in duration to a dotted quarter note.
2.2 Constructing scenes in the film

While the temporal length of individual positions in MFL was chosen by the tempo of the music, the film employed subject matter (propulsion), the framing of shots, lighting, and geometric symmetry to dictate content. Some of the scenes have apparent structural relationships, such as two different perspectives of the athlete and one from the front and one from the rear. Other relationships are subtle, such as that between the stationary shot of the dancer’s feet in the garage, framed in the background by the square house frame, bisected by a yardstick, and the man waiting for a train, shot from an angle under the platform, framed through a square in the foreground (the support structure of the station) bisected by stairs, for example.

2.3 Determining density of musical materials

After the themes were completed, the density of each location within the square was determined by reducing each number to modulus 3. This generates the following results:

| 0 2 0 1 2 1 |
| 1 2 0 1 2 0 |
| 1 2 1 0 2 0 |
| 1 2 1 2 2 1 |
| 1 2 2 0 2 0 |
| 0 2 0 1 2 1 |

Figure 4. The magic square of the Sun in modulus 3

The numbers with a remainder of 0 were then represented as 3. These numbers were correlated to the quantity of thematic ideas present in each square location. As there were not a proportional number of thematic combinations as there were for numbers with certain remainders, a Markov chain was employed to determine which themes would complete the square. This resulted in the following distribution:

<table>
<thead>
<tr>
<th>Position in route</th>
<th>1 2 3 4 5 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>One theme</td>
<td>Piano and electronics Piano Electronics Electronics Piano</td>
</tr>
<tr>
<td>Two themes</td>
<td>Piano Piano Piano and Electronics Electronics Piano</td>
</tr>
<tr>
<td>Three themes</td>
<td>Piano Piano Piano and Electronics Electronics Piano</td>
</tr>
</tbody>
</table>

Figure 5. The themes corresponding to the individual positions in the magic square of the Sun

2.4 Selecting routes for the music

Routes through the square were determined subjectively. The equal value of each of the routes enabled us to select routes independently and still have points where elements could be unified. After the musical themes were affixed to individual positions, eleven different routes through the square were selected, giving MFL a duration of just under nine minutes. Although we were both aware of these routes, we kept information regarding the correlation of specific themes or themes to the individual positions within the square separate.

2.5 Determining musical parameters

The quantity of thematic ideas in an individual square location as well as the position of the number in the route determined the instrumentation and orchestration of a particular thematic idea. If the third position in the route contained one thematic idea, for example, the instrumentation for that position would be piano alone. If there were two themes present, however, the position would be played by electronics alone. As seen in the grid in figure 9, the choices of instrumentation all follow this predetermined form.

Figure 6. Instrumentation as determined by its position in the route in MFL

Initial attempts to realize a version of the piece with an equal amount of each of the three possible options were made, but it immediately became apparent that this was not a viable option. First, the pianist would not perform during extended periods. Second, during the portions with three themes, the pianist would be forced to perform three complex themes simultaneously. Third, implementing this system would not be consistent with the balanced versus imbalanced paradigm already set in the themes and density within the magic square.

In addition to the choices of instrumentation determined by the square, the location of the number within the square also determined the type of compositional sieve employed, the type of digital signal processing (reverberation, spectral extraction, spectral dynamics, etc.), and dynamic shape of the individual location. The resulting modifiers ensured that while musical ideas were repeated, the combination of these elements was unique to each occurrence of individual locations within the magic square. As can be seen in figure 10, the listing of the different elements of the first route through the square of both the music and the film reveals the level of formalization of these components in the structure of the piece.
2.6 Organizing data

After the data was compiled for each position in the square and order of routes, a linear version of the piece was generated. In the music, this realization was constrained by a series of predetermined rules:

1. When a theme is presented alone, it begins on beat 1.
2. A theme must cycle through completely before beginning on beat 1.
3. When a theme is not played, it is suspended or paused. When it resumes, it begins on the next sequential beat.

These rules assured that the themes would all be presented in a method that does not favor a particular theme.

2.7 Structuring the film

While the film does not follow the same routes in the music, the first route in the film is the same as in the music. As seen in figure 8, the scenes in the first route adhere to the same one in the music.

The process for selecting routes and the manipulation of scenes, like the music in MFL, follows a structural methodology. This will be discussed in a more in-depth upcoming paper.

3 Presentation of Research

Since it completion, presentations of MFL at film festivals include screenings of the movie (with a recorded soundtrack) at the Berlin International Film Festival, ESPN’s EXPN Tube 2000 Film Festival, the Light Plays Tricks 3 Short Film Festival, on PBS, QVC, and other regional venues. In addition, MFL was the highest rated or best film of the year on internet movie websites beigeworld.com and zeroonefilms.com. It has also been presented as a live performance work at the SEAMUS 2002 National Conference, Electronic Music Midwest, the Walt Disney Concert Hall, and other regional venues. These performances were by several different pianists.

In addition to its presentation as a live performance piece and as a film, the music of MFL has been used as part of the film score for the independent feature film, Most

High. The film premiered at the Indiefest Film Festival in Chicago, where it won every award for which it was eligible, including the grand prize (the Vision Award) and the Sundance Audience Award. This was followed by a selection to the Hamptons International Film Festival where the film won the Golden Starfish, the largest independent film prize in the United States. It has since won the Atlanta International Film Festival. The director is currently receiving offers for theatrical and national DVD release by several major film companies.

4 Future Research

As all of the parameters of MFL are part of a formalized process, the addition of other parameters in MSC is a relatively simple process. In the next composition employing MSC, the addition of location as a formal parameter is the obvious next step. The current 5.1 encoding of multi-channel audio would provide six distinct possible outputs, providing 720 possible options. This only considers static locations. In order to apply a more formal approach, a system to determine perceived location would be constructed.

A longer format film or video is another option for the use of magic squares as compositional models. This would also provide and interesting study as to how the passage of time is perceived in magic square compositions when applied to longer structures. The use of magic squares in interactive or non-linear artworks is another possibility. As a route is played, the user could select the next route to be executed. This would generate a large number of possible realizations of the piece, while also providing a set number of variables at the local level (i.e. the themes would all remain the same).

Clearly, there are many possible directions and courses of action to pursue in the construction and realization of other magic square compositions. The complex nature of the magic square allows for many different approaches to create original artworks. As a larger body of works is completed, it will become more apparent which future course of study we will take.

5 References

www.maxopus.com