Modelling the Motivic Process of Melodies with Markov Chains

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Abstract

We present and evaluate a completion algorithm for partially known abstract motive sequences based on a Nested Markov Model. This completion method is integrated into the learning-based evolutionary environment MELOGENET for melody completion in order to reduce the search space when mutating motive sequences. In this way musically cohesive melodies are produced.

1 Introduction

Cohesion, a term used in text analysis, is a property of the text which gives it unity and makes it a whole rather than an arbitrary string of sentences put together. In our minds, cohesion creates coherence. Accordingly, in music, melodic cohesion is what gives a melody unity, makes it sound coherent and belonging to a certain musical style [1].

Although a melody is superficially described by low level entities like pitch and rhythm, it is the underlying structure of higher-order that captures the gist of a melody: distant patterns (for example motives) may be related by similarity or contrast; a process of tension and relaxation often unifies a phrase; melodic movement takes place on different levels of abstraction simultaneously.

The automatic invention of globally cohesive melodies is a complex task which has not been satisfactorily solved. As a long term goal, we propose to model melodic cohesion on the various structural levels such as interval, motive or phrase level separately and use the models for the generation of new melodies according to the principle of analysis by synthesis. The cohesion of the generated melodies allows to determine the relevance of the different components.

Computational models for automatic generation of melodies have a long history (for example [5], [7], [9] and [10]). Several systems make use of higher-level knowledge: In [3], melodic contour is locally improved by predicting motive classes with a fixed left context using a neural network.

In [6], motives are reproduced stochastically by using several independent Markov chains. A lack of global cohesion is observed in both cases.

In this paper, as a step towards a comprehensive model of cohesion, we focus on the impact of repetition and contrast of musical motives on melodic cohesion when generating melodies in a given style.

We present and evaluate a completion algorithm for partially known abstract motive sequences based on a Nested Markov Model. This completion method is integrated into the learning-based evolutionary environment MELOGENET [4] for melody completion in order to reduce the search space when mutating motive sequences.

2 The Motivic Process

Classification in music involves the segmentation of a piece and the categorisation of the segments (motives) depending on similarity-based criteria. Similar motives are grouped into motive classes. The motivic process of a melody is defined as a stochastic process which generates a sequence of motive classes (for example ababccab ). Since we are only interested in the relative position of the classes within a melody, we abstract from absolute class labels. For example ababccab is a template (abstract motive sequence) for both 13134413 and 73732273 given classes numbered 0 through 7.

When extracting higher-level structure from folk song style melodies, it is often sufficient to consider fixed length segments like motives of half
Table 1: Probabilities for the occurrence of a new motive in children, Shanxi and random abstract motive sequences.

<table>
<thead>
<tr>
<th>Position i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children melodies</td>
<td>1.00</td>
<td>0.87</td>
<td>0.30</td>
<td>0.37</td>
<td>0.53</td>
<td>0.23</td>
<td>0.57</td>
<td>0.37</td>
</tr>
<tr>
<td>Shanxi melodies</td>
<td>1.00</td>
<td>0.85</td>
<td>0.75</td>
<td>0.64</td>
<td>0.06</td>
<td>0.10</td>
<td>0.24</td>
<td>0.05</td>
</tr>
<tr>
<td>Random sequences</td>
<td>1.00</td>
<td>0.54</td>
<td>0.47</td>
<td>0.29</td>
<td>0.36</td>
<td>0.40</td>
<td>0.39</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 2: Correlation between the probability distribution for the occurrence of new motives in abstract motive sequences and two types of phrase structure.

<table>
<thead>
<tr>
<th>Phrase Structure</th>
<th>2+2+2+2</th>
<th>4+4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children melodies</td>
<td>-0.02</td>
<td>-0.15</td>
</tr>
<tr>
<td>Shanxi melodies</td>
<td>0.09</td>
<td>0.41</td>
</tr>
<tr>
<td>Random sequences</td>
<td>-0.16</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Note duration or changes of harmony on every quarter beat. However, the appearance of new motivic material is not necessarily tied to a regular segmentation of a melody. We illustrate this phenomenon for the example of phrase structure.

**Example**

88 Chinese Shanxi melodies from the city of Hequ and 30 German children songs have been selected from the Essen melody database [8]. They are written in 2/4-meter, 8 measures long, and their smallest metrical value is a 16th note. Abstract motive sequences are computed from these sets using the Ward clustering algorithm [2]. Additionally, 100 random sequences of length 8 are generated.

Phrasing information seems to be a styndependent feature: Although the sample melodies have been chosen according to the same musical features, they can almost perfectly be distinguished by their phrase structure. All selected German children songs consist of four 2-measure phrases ("2+2+2+2") whereas 86 of the 88 Shanxi songs under consideration are composed of two 4-measure phrases ("4+4").

Since we are interested in modelling the occurrence of new motives, the question arises whether one can take advantage of this homogeneous phrase structure. For a given style, we compute the probability that a new motive occurs at some position i in the abstract motive sequence (Table 1).

The correlation between these probability distributions and two types of phrase structure are given in Table 2. A significant correlation is observed between the Shanxi probabilities and the "4+4" phrase structure, reflecting the true phrase structure of the sample melodies. The result might be due to the fact that many selected Shanxi melodies consist of 4 measures which are repeated similarly or identically. For the children song style, the correlation values are too small to allow a hypothesis about the true phrase structure of the samples. In summary, the example shows that the occurrence of new motives cannot always be captured by modelling phrase structure.

**Figure 1:** Calculating the distance between abstract motive sequences ababccdc and ababbbcb: \( \text{xor}_{2,5} = 0, \text{xor}_{6,8} = 1 \).

In order to measure the degree of similarity between two abstract motive sequences \( M \) and \( L \) of length \( n \), we use a metric which has been introduced in [4]:

\[
\text{dist}(L, M) = \frac{1}{2n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \text{xor}_{i,j}(L, M)
\]

where \( \text{xor}_{i,j}(L, M) \) is true iff the relationships between positions \( i \) and \( j \) in the abstract motive sequences \( M \) and \( L \) are not equal (see Figure 1).

**3 A Nested Markov Model**

We propose a two-level Markov model to describe the motivic process. When listening to a melody, the occurrence of a new motive causes interest, whereas the repetition of motivic material consolidates the melodic process. Our model reflects this distinction by representing new motives by a first-order linear Markov chain. Every new motive is associated with a first-order Markov chain which is responsible for the arrangement of former motivic material. Mathematically speaking, this is a Hidden Markov Model where output probabilities have been replaced with Markov chains.

Figure 3 shows the structure of a three state Nested Markov Model for the German children song "Kreis, Kreis, Kessel"
4 Completion of Abstract Motive Sequences

The Markov model gives rise to a Local Markov Completion Algorithm (LMCA) for motive sequences in the style of a set of examples. The key idea consists in approximating the motivic process underlying the incomplete motive sequence by evaluating a suitable set of examples. The set consists of motive sequences belonging to the style under consideration which have minimum distance to the incomplete sequence (minimum distance environment). The transition matrices in the model are determined based on this environment. Finally, the most probable completion of the sequence is computed using dynamic programming.

In order to test the performance of the LMCA, more straightforward approaches are also considered: (i) Each unknown position of a motive sequence is filled with the locally most probable completion (relative to the minimum distance environment) in order to test the necessity of considering motive transitions. (ii) The usefulness of the (local) minimum distance environments is questioned by considering a Global Markov Completion Algorithm (GMCA), obtained from LMCA by computing the transition probabilities based on the whole set of examples instead of the minimum distance environments. (iii) To establish the relevance of the transition information contained in the set of examples, an even weaker Equidistributed Markov Completion Algorithm (EMCA) is defined by setting the transition probabilities to equidistribution. (iv) Finally, random completion is considered.

Experiment

The completion performance of the LMCA and the four straightforward algorithms is tested on the data used in the example in section 2. The set of examples are split randomly into a test and a reference set. A number of positions is deleted at random from every abstract motive sequence in the test set which is then completed using the various algorithms and style information from the reference set. For every algorithm, the percentage of correct reconstructions and the average distance between the completed motive sequences and the original sequences are computed.

On the large and homogeneous set of Shanxi melodies, the local completion (i) performs best (Figure 3). Local and Global Markov completion produce similar average distances between the reconstructed and the original motive sequences. This finding gives rise to the supposition that for very homogeneous sets of motive sequences, minimum distance environments converge towards the whole set of examples.

On the smaller and less homogeneous set of children songs, the Local Markov Completion Algo-
rithm exhibits better behaviour than the other algorithms (Figure 4). Especially for up to three deleted positions, the Local Markov Completion is able to take advantage of context information and find completions with small average distance to the original.

5 Melody completion

![Melodies generated by MELOGENET](image)

Figure 5: Melodies generated by MELOGENET, above with and below without abstract motive sequence (children song style).

We incorporate the Markov completion algorithm into the learning-based evolutionary environment MELOGENET which is able to complete the beginning of a melody in the style of a set of examples. MELOGENET gradually improves a population of melodies in an evolutionary loop. The melodies are rated using neural networks which have been trained with examples of a given style. In this framework, the Local Markov Completion Algorithm is employed to reduce the search space when mutating motive sequences by retrieving the musical background knowledge stored in the transition probabilities [4]. Figure 5 shows two melodies produced by MELOGENET. The first melody was generated using the Local Markov Completion Algorithm, the second without consideration of the abstract motive sequence. Although both melodies exhibit typical motivic material, the arrangement of motives is better for the first melody which sounds more cohesive.

6 Conclusion

We have given a musical example illustrating that MELOGENET yields more cohesive melodies when using an abstract motive sequence than without it. It was shown that the correlation between phrase structure and the occurrence of new motives in an abstract motive sequence is style-dependent. For small, inhomogenous sets of abstract motive sequences, the Local Markov Completion Algorithm produces better completions than straightforward algorithms. On large homogenous sets, the simpler Local Completion Algorithm seems to be more appropriate. Future research will be concerned with integrating phrase structure information and the occurrence of new motives into one model of abstract motive sequences.

References


