Abstract

The computer study of music began in the late 50's. Yet, there is no general computer research on music, and existing works do not provide any stable basis for further research. What is missing is a formal basis for the computer modeling of music theory. Two main subjects in the development of such a basis are the selection of primitives and of a representation language. A computer system, called CSM (Computer Study of Music), that is based on a formal study of these two subjects for Western Tonal Music (WTM) was built. The ultimate goal of this system is to lay a "corer stone" for a general formal study of WTM. This paper summarizes the theory behind CSM, its implementation, and our plans for future research.

1. Introduction

The study of music (especially WTM) provides an extraordinary domain for an Artificial Intelligence (AI) research. A field of intense human activity, with an abrrovix theory, but with a large (unformal) theoretical basis and with criteria for performance evaluation, etc., thirty years of work on computer analysis of music did not initiate any general research on computer modeling of WTM theory. Moreover, existing works do not provide any stable basis for further research. Specific works with weak compositional basis and unclear theoretical content cannot be extended to handle larger areas.

Artificial intelligence, backed by its computational methodology for handling abstract knowledge, provides a most promising approach for building a formal basis for music research. Accordingly, both fields are likely to benefit from a marriage of music and AI research.

Looking for a firm unified computational basis to support representation, processing and possibly construction of musical theories, we started the work with a stock of appropriate formal mechanisms for the computer processing of WTM. Two low level "primitives" standing for the musical concepts of Note, Interval, and the syntactic aspect of Twelve-Tone-Structure were employed as building blocks for the common terminology of WTM. The study of appropriate representation forms first implied the inadequacy of known grammatical models. The suggested formulations are developed in two successive abstractions, beginning from a conceptual framework consisting of terms, operators, attributes and relationships, through its abstraction as a set of units called Generalized-Concepts (GC-C's), into a further abstraction as a set of constraints systems.

This theoretical basis was implemented in a PROLOG system, called CSM, that supports a general study of WTM. The kernel of CSM's knowledge base contains of the core concepts of WTM theories, formalized within the GC model with a first order predicate language, and it can be easily extended to handle real theories of WTM.

In what follows, we discuss the selected primitives, the GC model and CSM's control. (For more detailed description see [Bulian 82, 83, 84, 85].)

2. Background

2.1 Computer study of music

The beginning of computer music and computer minded music research is marked by the pioneering work of Holzer and Inbosen in the 50's. The early works focused on the application of either numerical or probabilistic or statistical, or information based methods to musical data bases. (Eliot & LaVaton 1956, Brooks et al 1967, [Pinkofsky 1962, Lincoln 1970].) The main weakness of these systems was their expressive power, which was limited to that of finite automata. This period in the study of music, reminds the failure of the "mechanical translation" of natural languages, which was based on similar methods and happened in about the same time.


Most works on the computer study of music focus on WTM. They share three important properties: the musical scope is quite restricted, they do not claim the ability to deal with music beyond the Western musical tradition and, for reasons of time and expertise, the computer calculations are used in instrumental work or in a music oriented environment. There is no study of computational formalisms per se. In particular, there is no general, formal, computer study of WTM.

2.2 WTM Theory

Though, there are specific, domain restricted theories that admit formal computational models (e.g. Schenker's and more recently Narmon's "Implication - Realization" model [1972], and Lenthak and Jankowski's "Generative Theory of Tonal Music" [1985], there is no generally accepted, well defined theory of WTM. Therefore, a theoretical study of music is a study of "human activity" which lacks a completely organized theory, i.e., a typical area for an AI researcher.
2.3 WTM and Natural Language Processing (NLP)

One basic similarity of WTM to natural languages is that people from different cultures have an intuitive, unconscious (unstudied) knowledge of WTM, knowledge which enables them to accept, reject, or analyze infinitely many new material features. This similarity makes WTM especially appropriate for a tectonic research since: (i) It provides an evaluation criterion, i.e., the judgment of the "experienced listener"; and (ii) it contains a "language space" requirement, i.e., a suggested model should be able to account for indefinitely large numbers of structures.

Considering these and many other similarities between WTM and natural languages, and in view of the intensive research on NLP, the current state of computer study of music seems quite surprising. One is inclined to say that since both NLP and the computer study of music went through a statistical-numeric period in the 50's, followed by a syntax free period, it is possible that the computer study of music lacks some parallel to the current "semantics first" period of NLP [Milner 1971; Meessen 1980]. [Luske 1982] and [Radin 1982], all emphasize the inadequacy of many oriented music theories in several basins for a music understanding research, and neglect the AI approach as more appropriate.

2.4 An AI Approach

AI systems are often called "open ended" systems since their main characteristic is that they are not intended to solve specific, well defined algorithmic tasks. Rather, they handle incomplete, ill-defined problems, the knowledge about which is ever growing, ever changing. A typical AI system if there is such a thing, must contain a lot of knowledge about its subject domain, and must be provided with means to manipulate this knowledge. The knowledge how constitutes a representation of that part of the world that is assumed relevant to the AI system.

The AI system is then taken to account for that knowledge, and the mechanism used for reasoning determine the power of the system. For example, natural language systems are adequate at the point of view of expressive power and user convenience. However, they are an uncooperative representation formalism, and their manipulation is hard. On the other extreme, the computer science of representation and conventional language for stating exactly which notes should be played, at what point, etc. as a representation language it is too restricted. Partially specified pieces, and meta properties of a piece like its structure or its role, cannot be represented in this language. Referring to the role of representation in AI, Woods says in 1980:

"In computer science, a good solution often depends on a good representation. For most AI applications, the problem is of representation is even more difficult, since the possibilities are so much greater and the criteria are less clear. The choice of representation becomes crucial in the choice of reasoning and knowledge of intelligent agents that can understand natural language or character percep- tual data, because the representation is the primitive of and the system for their combination effectively limit what such systems can ver- tify, know, or understand."

Knowledge representation and reasoning mechanisms are, indeed, the key issues of AI.

One source of difficulty in designing AI systems is the lack of clear definitions. There is no well-defined domain. Compare, for example, the following two problems:

1) Sort a list of numbers, e.g., the list (1,2,3,3) should be transformed into the list (1,2,3,3).
2) Design a system that is capable of analyzing music according to a given theory, e.g., if the theory is "in Mozart music half of the chords are Seventh chords" then the system is able to test Mozart pieces that new claim.

In the sorting problem, the subject domain is clear, and no one would want to sort a list of chords by a production list that should be sorted, or that only four elements lists should be sorted. The difficulty here is in the stage of problem formulation, but in designing an appropriate algorithm. The music problem, on the other hand, is vaguer stated. Clearly, if we design a computer program just for computing the percentage of chords in a piece we may criticize a production since Schaeffer's theory is also an incense of this problem, as well as all the rules appearing in textbooks of music theory. Is the subject domain, i.e., music theory that should be explained before we go on with the system design. The music problem, as stated now, is exactly the ultimate goal of the research described in this paper.

The explanation of the subject domain usually imposes questions like:

"What part of the world is relevant to the problem?"

"What general properties can be assumed so that no instance of the problem is ruled out?"

"What are the required properties of a representation language?"

"On what level should the knowledge be represented?"

or, "What are the representational primitives?"

"How to manipulate the knowledge about the world?"

Questions of this kind are hard, and there hardly any theory about how to produce good answers. The theory of AI provides, at least, certain guidelines for evaluation of AI sys- tems, but the process of designing good AI systems is still a muster of art.

2.5 History of this project

This research grew out of disappointment from an experiment in which a computer program was designed to test us to listen to a sequence of notes and to associate with [Schonberg 1910; [Feldman 1975]; [Radl 1980]. While the musical results were quite satisfactory, it was clear that the representation used for this theory, i.e., a pro- gramming language, severely restricts the power of the system, especially in terms of polyphony and general- lization. What was missing was a firm conceptual base that can support research in designing and even construction of musical theories. Existing works could not be of much help since their computational bases was weak in many important respects, like expressive power, simplicity, generalizability and extensibility. In partic- ular, works on different subjects cannot be combined in a simple way. Consequently, we decided to start a new project, with the ultimate goal of providing a formal basis for the study of music theories.

The questions that we faced at the beginning were exactly the sort presented in the previous section. At first, we set down to characterize the basic objects, in terms of which we expect music theories to be stated. This was the development of the stages. The next step involved selection of a representation language. These two steps are the basis for the CSM system.

3. Level of Representation

Observing that WTM does have a common terminology in statement, descriptions, and theories of WTM, it seems natural to take "the common
denominator type" concepts of WTM's theories as the
desired level of representation. That is, the common ter-
mology found in text books was suggested as the vocab-
ulary for representing WTM’s theories.
The cost of the relatively coarse representation for
music researchers, is the need to develop a precise basis
for the high level vocabulary, to enable the processing
of the represented material. We found this to be a rather
complicated process since it is not clear what kind of
object is a musical concept, it is not clear what concepts
should be considered as being of the "common denomi-
nator type"; and music text books never lacked to
provide precise definitions to basic concepts mainly
because these concepts are clear on the intuitive level.
These difficulties imply that the set of basic concepts
should be studied as if it was also a musical theory.
Consequently the process of establishing a high level
vocabulary was split into three steps: (a) study of
appropriate low level representation; (b) Study of the
common musical terminology in terms of the low level
concepts. The result was stating the Skeletal Theory of
WTM. (c) Embedding the Skeletal Theory in an
"appropriate" representation formalism.
3.1. Low Level Representation
A low-level basic concept is a concept that
represents a "new" thing; that is, is not a specialization
of other concepts appearing in music theories and
its entities are not coming composite structures (like
lists, sets, etc.) of entities of other known concepts.
We expect that a considerable number of standard
WTM concepts would be definable in terms of the
selected low level "primitives", using operations over
common domains like arithmetic, strings, finite lists,
etc. The definition of a concept in terms of the "primi-
tives" is not expected to be unique since it seems that
the existence of several meaningful analyses to a musical
structure of concept in a basic characteristic of music.

4.11. Notes, Intervals
The physical elementary object of music is the
metrical time. The notion of duration is inferred.
The main problems in describing these concepts in WTM
are their multifaceted appearances in musical pieces (the
rhythmic feature), and the ambiguous usage of each
concept. These problems tend to create confusion, un-
reliable ambiguity, and misunderstanding. It is impos-
sible that a major portion of any work attempting to
develop a formal and computer implemented study of
WTM would be devoted to resolving these difficulties.
We tried to arrive at the "best" meaning of each of the con-
cepts of Notes and Intervals into several unambiguous
sub-concepts, and explaining the interconnections
between the new subconcepts. This study led to the
development of a unified representation, entitled Prim-
itive Elements, for the theoretical terminology of Notes
and Intervals.

Notes The set of physical notes of WTM (assumed
unbounded from both directions)1, can be mapped onto
the integers, by a straightforward, one-to-one
enumeration, mapping the piano middle C to zero, notes above it
are positive integers; and notes below it are negative integers.
The common tone scale terminology for

1 We claim that the bounds on our set are not intrinsic to the human
sensibility. As WTM is a model for music, WTM should not rely
on these. This assumption, yielding infinite sets, simplifies the discussion
of theoretical notes, representing the expanding total models of music
and enables the development of a unified representation for Notes and
Intervals.

notes, entitled theoretical-notes names consists of two
components an octave transfer and a name of a pitch
class, which in turn consists of distance and alteration
components. The intended physical meaning of the
theoretical-notes names (i.e., the denoted physical notes)
can be obtained by a component-wise, many-to-one map-
ing, called here ATN (Absolute Translation of Notes).

Intervals The set of directed intervals in WTM2
can also be mapped onto the integers, by a straightforward,
one-to-one enumeration, associating an interval 1/2/3
with the integer n. The common tone scale terminol-
ogy for intervals, entitled theoretical-interval names,
consists of three components: A direction, a non-negative
octave number, and a name of intervals which in turn
consists of a basic group (like PRIME, SECOND, etc.),
and a shift (like MAJOR, MINOR, etc.). The intended
physical meaning of the theoretical-interval names (i.e.,
the denoted physical intervals) can be obtained by a
component-wise, many-to-one mapping, called here ATI
(Absolute Translation of Intervals).

Primitive-Elements The similarity between the
theoretical terminologies for notes and intervals, both in
structures and intended meaning, suggested the idea of a
unified numeral representation entitled Primitive-
Elements Table 1, below, presents the primitive-
 elements representation of several theoretical-notes names
and theoretical-interval names. Figure 1, describes the
sub-concepts mentioned so far, and their relations to the
synthetic Primitive-Element concept.
The primitive-elements preserve the structure and
the intended physical meaning of the theoretical ter-
mologies since the mappings ATN, ATI are component-
wise defined, and a theoretical-note-name and a
theoretical-interval-name that are represented by the
same primitive element, have the same intended physical
meaning.

| theoretical- |
| notes names |
| theoretical- |
| interval names |
| primitive-element |
| [1D,2D] | (UP,1) | SECOND, AUG(2) |
| [1D,0D] | (DOWN,1) | SECOND, AUG(2) |
| [1D,2D] | (UP,0) | PRIME, PERFECT |
| [1D,0D] | (DOWN,0) | PRIME, PERFECT |

Table 1 Primitive-elements representation of theoretical-
 notes names and of theoretical-interval names.

The Primitive element concept enabled the develop-
ment of a simple algorithmics, using one additive opera-
tion, which can account for all common computations
involving the WTM terminology for Notes and Intervals.
In addition, another structure, basis chords and predic-
tives, which can be interpreted as either structures of
notes or structures of intervals can be described by a con-
crete structure of Primitive Elements. Consequently, Primi-
tive Elements were selected as Low Level "Primitives",
standing for both Notes and Intervals.

2 A modulo interval is opened directed if the first note is higher than the
second, and is closed otherwise. A harmonic interval is considered,
by agreement, as closed directed.

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3.12. Twelve-Tone Pieces - Syntactic aspect

Consider the following "piece":

\[
\begin{align*}
\text{C} & \quad \text{in (C, } \frac{1}{2}\text{)} (C, \frac{1}{4}) \quad \text{D} \\
\text{E} & \quad \text{in (E, } \frac{1}{4}\text{)} (D, \frac{1}{4}) (C, \frac{1}{2})
\end{align*}
\]

The notes can be represented by the primitive-elements as in figure 2.

\[
\begin{align*}
\text{E}_0 & \quad \frac{1}{3}\text{G}_0 \quad \frac{1}{3}\text{F}_0 \\
\text{D}_0 & \quad \frac{1}{2}\text{C}_0 \\
\text{C}_0 & \quad \frac{1}{2}\text{G}_0 \\
\text{G}_0 & \quad \frac{1}{2}\text{B}_0 \\
\text{B}_0 & \quad \frac{1}{2}\text{E}_0
\end{align*}
\]

Another representation for that piece is:

\[
\begin{align*}
\text{E}_0 & \quad \frac{1}{3}\text{G}_0 \quad \frac{1}{3}\text{F}_0 \\
\text{D}_0 & \quad \frac{1}{2}\text{C}_0 \\
\text{C}_0 & \quad \frac{1}{2}\text{G}_0 \\
\text{G}_0 & \quad \frac{1}{2}\text{B}_0 \\
\text{B}_0 & \quad \frac{1}{2}\text{E}_0
\end{align*}
\]

\[
\begin{align*}
\text{E}_0 & \quad \frac{1}{3}\text{G}_0 \quad \frac{1}{3}\text{F}_0 \\
\text{D}_0 & \quad \frac{1}{2}\text{C}_0 \\
\text{C}_0 & \quad \frac{1}{2}\text{G}_0 \\
\text{G}_0 & \quad \frac{1}{2}\text{B}_0 \\
\text{B}_0 & \quad \frac{1}{2}\text{E}_0
\end{align*}
\]

Then a note with its notation can be represented as a pair. For example, \((C, \frac{1}{4})\) denotes \(\text{C}_{\frac{1}{4}}\).

If we consider the piece as the simultaneous performance of two melodies it may be represented as:

\[
\begin{align*}
\text{Soprano} & \quad \{ \text{E}_{\frac{1}{4}}, \text{D}_{\frac{1}{2}}, \text{C}_{\frac{1}{2}} \} \\
\text{Bass} & \quad \{ \text{E}_{\frac{1}{4}}, \text{D}_{\frac{1}{4}}, \text{C}_{\frac{1}{2}} \}
\end{align*}
\]

And bass is \(\{ \text{E}_{\frac{1}{4}}, \text{D}_{\frac{1}{4}}, \text{C}_{\frac{1}{2}} \}

That is, the piece is represented as the horizontal combination of two horizontal structures.

Taking the piece as successive performance of intervals may be represented as:

\[
\begin{align*}
\text{EC} & \quad \text{DG} - \text{CC} \\
\text{DG} & \quad \text{EC} \quad \text{EC}
\end{align*}
\]

That is, the piece is represented as the horizontal combination of vertical structures.

This notation may be further generalized to represent any twelve-tone score. This is the TTS notation described below.

The term musical piece corresponds to the terms string and sentence of formal and natural languages. We can theoretically define them as a formal object based on the theoretical terminology for notes or intervals augmented with a rest symbol, accents for the duration property of the denoted physical objects, and the indications between necessary and optional appearances of the physical objects. In addition we want this term to account also for structural entities, i.e., the existence of several meaningful decompositions of a musical piece. It seems that achieving structural unambiguity is vital for any attempt made to formally deal with more advanced issues concerning musical pieces.

A representation, entitled Twelve Tone-Strings (TTS's), was developed along these lines. An elementary TTS is a piece of a primitive element or a rest symbol) and its duration. A TTS is either an elementary TTS, or a structure \((\alpha, \beta)\) standing for the concatenation of the pieces represented by the TTS's \(\alpha\) and \(\beta\) at the same time and at the beginning of the piece represented by \(\alpha\). A TTS representation of a musical piece reflects, in a simple and natural manner, the intended structure.

Example: The TTS

\[
\{ (p_1, 1), \{ (p_0, 1/4), (p_1, 1/2) \}, (p_2, 1/2) \}
\]

where \(p_0, p_1, p_2\) are \(0, 1/4, 1/2\), respectively.\(p_0, p_1, p_2\) are \(0, 1/4, 1/2\), respectively. Each \(p_i\) denotes \(\{ p_0, p_1, p_2\}\), i.e., simultaneous appearance of the piece.

TTS-Representation of some typical musical structures:

1. Harmonic sentence of ioved chord:

\[
\begin{align*}
\{ & \text{p}_0, 1 \} \quad \{ \text{p}_0, 1/4 \} \quad \{ \text{p}_0, 1/2 \} \\
& \{ \text{p}_0, 1/4 \} \quad \{ \text{p}_0, 1/2 \} \quad \{ \text{p}_0, 1/4 \}
\end{align*}
\]

where all \(p_0\) are elementary TTS's. This is a horizontal concatenation of vertical concatenations of elementary TTS's.

2. A melody with an accompaniment:

\[
\begin{align*}
\{ & \text{p}_0, 1 \} \quad \{ \text{p}_0, m/4 \} \quad \{ \text{q}_0, 1/4 \} \\
& \{ \text{p}_0, m/4 \} \quad \{ \text{p}_0, m/2 \} \quad \{ \text{q}_0, 1/4 \}
\end{align*}
\]

where \(q_0\) is a rational number and all \(p_0\) and \(q_0\) are elementary TTS's.

This is a melody, concatenated vertically to its accompaniment.

3. A "polyphonic sentence", i.e., a sentence obtained from the simultaneous appearance of several melodies.
where all $p_k$ are elementary TTS's. This is a vertical concatenation of melodies.

Note: The TTS representation provides a convenient method for inputting melodic information involving only the twelve tones system. Its major advantage over other existing input languages (e.g., DARMES, see Erickson [1977]) seems to be its simple modular structure.

4. Representation Forms

A formalism designed to support a general formal research of music should possess properties such as being powerful, uniform, simple and invariant. An immediate implication is the inapplicability of known grammatical models. This can be based on common arguments of expressive power and simplicity, but the main argument is that from the semantic point of view, the structurally ambiguous forms are usually interfaced. A similar claim was made by Narmour in [1977].

4.1. The Generalized Concept (G-C) Model

This formalism for the representation of musical theories was developed in two successive stages: beginning from a conceptual framework consisting of concepts, attributes and relationships, through its abstraction as a set of units called Generalized-Concepts (G-C’s), into a further abstraction as a system of constraints.

4.1.1. Conceptual Framework

We assume that musical theory is a set of musical hypotheses; each hypothesis has a theme which might be an abstract musical concept like chord-base — the set of note-names (frame of pitch classes) of a chord, a physical musical concept like a melody, a relation between musical objects, a particular instance of a musical concept, etc. Therefore, a musical theory is a set of instances of musical objects.

We assume two kinds of conceptual objects: concepts and relationships.

Concepts represent collections of distinguishable musical entities that have similar structure or content. The information content of a concept is represented by a finite number of mappings from, or relations between, the concept to other domains. These mappings/relations are called attributes. Each attribute is associated with one domain, i.e., a set of values. The instance and attribute-value assignments of a concept admit some requirements which can be finitely described by conditions to be satisfied. These conditions may involve other concepts and these attributes. For example, the Terzain-chord concept may be described as the set of TTS's instances of kind vertical, which satisfy a certain "set-theoretical" condition.

Relationships are relations among concepts, or relations on a set of concepts. They also may have attributes. Examples are the binary set relationship, the concept-of relationship and classification relations. An important family of relationships among concepts is the frame-of-reference relationship, relating a concept to a possible environment or context to another concept(s).

For example, the Scale concept is a frame of reference to the Note-name concept (a name of a pitch class like $G, b$). (C.B.) assigning it a degree and pitch attribute. Figures 4 and 5 describe two conceptual descriptions, using graphical notation. Boxe denote concepts, circles denote attributes-domains; labeled arcs represent attributes; labeled double arcs represent relationships involving the concepts at the arc ends.

![Diagram 4](image4.png)

**Figure 4** The Primitive-element-name concept, i.e., the "basic-group-shift" component of primitive elements, that stands for names of pitch-classes (like $G, b$), and for names of intervals (like PRIME-PERFECT).

![Diagram 5](image5.png)

**Figure 5** The frame of reference relationship between the Note-name concept (All entities in Primitive-element-name that have notes meaning) to the Scale concept. (A scale is a particularly structured list of entities of Primitive-element-name) Attributes of related concepts that are relevant for the modeling of this relationship (see example below) are also shown.
4.1.2. The G-C representation model

The structure or concept of a concept can be con-
structed (see section 3.1.1) by the relationship among the values of the concept's attributes. A concept C and its attributes can be represented as a 2-tuple C = (A, C), where A is a set of attributes and C is the value of the concept.

Example: Note-name-over-a-scale

This G-C describes the frame-of-reference relationship between the Note-name and Scale concepts, which was conceptually described in section 3.1.1. It has attributes for Note-name, Scale, and for the relationship.

attributes: Note-name / Scale

Note-name: stands for the note-name
Scale: the diatonic component
ALTERATION: the alteration component

The relationship described by this G-C is the relationship between Note-name and Scale.

scale - a scale used in a frame of reference to note-name

Note-name: the note-name of the scale
Scale: the scale used in the relationship
ALTERATION: the note-name of the scale

scale-alteration: the alteration of the note-name with respect to scale.

relationships (the A-s)

Note-name-over-a-scale: stands for the G-C NOTE-NAME-OVER-ASCALE
PRIMITIVE-ELEMENT-NAME: stands for the G-C PRIMITIVE-ELEMENT-NAME
SCALE: stands for the Scale G-C.

left-hand side:

NOTE-NAME-OVER-ASCALE (note-name, distance, alteration, val, musical-representation, scale, scale-degree, scale-alteration)

Constraints (the A-s): There are four constraints: Constraints (1) and (2) are "is-a" constraints, defining the relevant attributes of Note-name and Scale. The third constraint defines the relationship's attributes.

1. note-name, diatonic, alteration, val, and musical-representation form an element of the PRIMITIVE-ELEMENT-NAME G-C, with the value NOTE for the type attribute.
was developed, using first-order predicate logic language. This step completes the process of establishing the high level vocabulary.

6. CSM

CSM (Computer Study of Music) is a PROLOG system designed to support a general computer study of music. It is able to do analysis, partial analysis and synthesis. The PROLOG language was selected for its close relationship to the G-C formalism, i.e., a D-G-C can be taken as a PROLOG rule of the form
\[ \text{C}(a_1, \ldots, a_n) \text{ if } A_1(a_1, \ldots, a_n) \ldots A_m(a_1, \ldots, a_n) \]
\[ \text{G}(a_1, \ldots, a_n) \text{ if } B_1(b_1, \ldots, b_n) \ldots B_m(b_1, \ldots, b_n) \]
\[ \text{C}(c_1, \ldots, c_n) \text{ if } D_1(d_1, \ldots, d_n) \ldots D_m(d_1, \ldots, d_n) \]

CSM has two main component: a knowledge base of D-G-Cs, and a control component.
The kernel of the knowledge base is the D-G-C-T for the Skeletal Theory of WTM. Every D-G-C is formulated by several PROLOG rules. For example, a simplified version of the PROLOG rules for the D-G-C that describes the TTS concept is as follows:

\[ \text{tts-concept}(\text{ts}(\text{ThIs} \{\text{ThIs2} \ldots \text{ThI} \ldots \text{ThI} \}) \text{kind: horizontal}, \text{L simplified form: ThIs} \ldots \text{ThI2} \ldots \text{ThI}, \text{duration: D}, \ldots) \]
\[ \text{tts-concept}(\text{ts}(\text{ThIs} \{\text{ThI} \}) \text{kind: horizontal}, \text{L simplified form: ThIs} \ldots \text{ThI2} \ldots \text{ThI}, \text{duration: D}) \]

and similar rules for the elementary, vertical, and the general case. This rule says that \( \text{ThIs} \ldots \text{ThI} \) is a horizontal TTS whose simplified form is \( \text{ThIs} \ldots \text{ThI2} \ldots \text{ThI} \), and its duration is \( D \) and \( \text{ThIs} \ldots \text{ThI} \) are both TTSs, and Time is the duration of \( \text{ThIs} \ldots \text{ThI} \).

Analysis of, for example, the TTS
\[ \{(0,1.0), 1/4 \} \{1.0, 1.25, 1.5, 1.75 \} \{1.25, 1.5, 1.75, 2.0 \} \]
that stands for the interval \( \{0, 1.5, 2.0 \} \), can be obtained by presenting it as an interval of TTS. The appropriate rule would be applied, and would select values to the kind, simplified form and the duration attributes. Synthesis of a horizontal TTS of duration \( D \) can be obtained by presenting the goal \( \text{tts-concept} \) \( \text{ts} \{\text{ThIs} \ldots \text{ThI} \} \text{kind: horizontal}, \text{L duration: D} \).

Again, the rule that would be applied would select values for the tts and the simplified form attributes.

Unfortunately, the PROLOG formulation of D-G-C is not straightforward. In fact, a good PROLOG formulation requires deep understanding of the interconnections among the attributes of the D-G-C, and a deep understanding of PROLOG. Good formulation is crucial for a system like this since inefficiency in processing the basic concepts would doom the whole system. A special difficulty results from the absolute multidirectional nature of the task. That is, analysis, partial analysis and synthesis, all should be handled by the system. A former implementation that tried to split among these cases failed due to an explosion in the number of cases that had to be handled. The special control strategy used by CSM partially solves this problem.

The control component is responsible for processing the D-G-Cs of the knowledge base. The PROLOG
control is of no use since it is a fixed control, that always selects the first goal in the list of goals, and applies the first applicable rule. In a totally unordered system like the D-G-C's knowledge base, the order of processing the conditions in a D-G-C should be flexible. For example, if in the 
Note-name-override-scale example in section 4.1.5, a given partial interval is:

\[
\text{scale} = \{ (1.0, 1.0), (1.0, 1.0), (0.9, 1.0), (1.0, 0.9), (1.0, 1.0) \}, \text{scale-degree} = 0.5
\]

then the constraint, which accounts for the connection between distance to scale-degree, this should be resolved first. Starting with the first constraint, we might end up with many useless selections of value for static. The control component consists of rules about how to process the rules of the knowledge base. These rules use different criteria to answer the question: "Given a set of goals, G1, . . . , Gn, what is the next goal to be resolved?". The control component consists of rules about how to process the rules of the knowledge base. These rules use different criteria to answer the question: "Given a set of goals, G1, . . . , Gn, what is the next goal to be resolved?"? Barely, the goals G1, . . . , Gn, would be the conditions of the D-G-C under consideration (The A-ar). For example, a rule for handling conditional dependencies, together with the information that in the Primitive-Name-override scale degree of the G-C, the basic-group attribute functionally depends on the primitive-name-override attribute, would do the work in the example above. This control strategy also simplifies CSS since the D-G-C's can be free from interest information.

7. Conclusions and Future Research

As far as we know, CSS is the first computer system built to support a general computer study of WTM. It is based on a theoretical study of primitives and representation forms. The suggested G-C model was used to formulate the core concepts of WTM theories. Most of these are already implemented in CSS.

We believe that both the G-C model and the output strategy have applications outside the music domain. The model can control the declarative power of frames with the inferential power of logics. The control strategy is a relatively efficient human oriented reasoning strategy.

Our plans for the future are to apply CSS to several real theories. Two such candidates are the harmonic progression hypothesis of Schoenberg (1954), Uses of the Heuristic (Lerdahl) and Jackendoff's theory (1983). It is expected that a representation language that is richer than first order logic would be required, in order to express, for example, deniability and likelihood.

References


