Estimation Of Parameters Corresponding To A Propagative Synthesis Model Through The Analysis Of Real Sounds

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Abstract

Simulation of sounds produced by musical instruments can be achieved by the use of synthesis models that take into account the wave propagation in the medium. Here we address the problem of the estimation of parameters for the simple waveguide model. The filter and delay inside the loop mainly take into account phenomena linked to the propagation (dispersion, losses), while the filter outside the loop takes into account the excitation and radiation aspects.

In order to estimate these filters, we propose an approximation of the global transfer function of the system in terms of a sum of independent partials with exponential decay, the amplitude, frequency and bandwidth of which can be directly related to the characteristics of these filters.

In the case of plucked string instruments, or flute, excited with a transient, the bandwidth of each component is related to its amplitude modulation law in the time domain. By the use of time-frequency techniques, we estimate the parameters corresponding to the waveguide model, and compare the datas with the theoretical values coming from the solutions of the beam movement equations.

I) Introduction

Waveguide models make the simulation of propagation of acoustical or mechanical waves in bounded media possible. From a theoretical point of view, it is possible to deduce the parameters of the model from the solution of the movement equations. Nevertheless, these equations are often resulting from approximations, yielding a biased estimation of the parameters. In this paper, we address the problem of the estimation of the model parameters from the analysis of real sounds, and the matching of the measured values with the theoretical ones. In the case of the flute and the guitar, relevant parameters such as dispersion and dissipation can be estimated both by studying the solutions of the movement equations and by the analysis of real sounds. The dispersion phenomenon introduces an inharmonicity between the partials, and the dissipation yields a different decay time of the components. These two effects can be measured on real sounds. In order to use the waveguide synthesis model in practice, we shall propose a way of building the filters of the model that will reproduce the dispersion and dissipation phenomena.

II) Movement equations

We are working on the propagation of longitudinal waves in fluids (wind instruments) and transverse waves in solid media (string instruments).

II.1 - Movement equation in a tube

When visco-thermal losses are taken into account, the propagation equation of the pressure can be written as [1]

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial}{\partial x} \left( \frac{1}{c} \frac{\partial p}{\partial x} \right) = 0$$

where $h_{av}$ represents characteristic lengths in the air ($h_{av}$ is of order $10^{-1}$ in a free field). Its equivalent value in a bounded media like a tube depends on the friction near the body of the instrument, and cannot be easily determined.

The solution of the wave equation for a tube which is open at both ends (flute) and submitted to a punctual disc source can be calculated using classical techniques [2]. It can be given by a superposition of components, each of them being given by

$$p_0(x,t) = \frac{1}{2} \left( \frac{\sin \left( \frac{x - x_0}{L} \right)}{\sin \left( \frac{\lambda u_0}{L} \right)} \right) \sin \left( \omega t \right) e^{j\omega t},$$

where $L$ is the length of the tube and $x_0$ is the source position.

The frequencies are given by

$$\omega_0 = \sqrt{\frac{c^2 \lambda L}{L^2}} - \frac{1}{2} \lambda \sqrt{\frac{c^2 \lambda L}{L^2}} - \frac{4 \lambda^4}{4 L^4},$$

where the inharmonicity depends on the value of $h_{av}$. 

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The damping factors are given by:
\[ \alpha_n = h_b \omega_n \left( \frac{\omega_n^2}{\omega^2} - 1 \right) \]

This model can lead to both physical and additive synthesis models [3].

II.2 - Movement equation in a string

The transverse displacement \( y(x, t) \) of a stiff string is described by the equation [4]:
\[ \frac{\partial^2 y(x, t)}{\partial t^2} - \frac{1}{\rho} \frac{\partial}{\partial x} \left( \frac{\rho \partial y(x, t)}{\partial x} \right) + \frac{\rho}{2} \frac{\partial^2 y(x, t)}{\partial t^2} = 0 \]

If we assume that the boundary conditions are:
\[ y(0,t)=y(L,t)=0, \quad \frac{\partial y(x,t)}{\partial x} \bigg|_{x=0}=\frac{\partial y(x,t)}{\partial x} \bigg|_{x=L}=0 \]

we can show that the transverse displacement of vibration can be written as:
\[ y(x,t) = \sum \frac{\sin \left[ \frac{n \pi x}{L} \right]}{n \pi} e^{i \omega_n t} \]

with \( \omega_n = \sqrt{\frac{2}{\rho} \frac{\pi^2 n^2}{L^2} - \frac{2}{\rho} \frac{\pi^2 n^2}{L^2}} \)

III - Waveguide model

In order to simulate the stationary waves corresponding to the propagation in a bounded medium, we consider the following model [5]

![Fig.1 Waveguide model](image)

To match this model with the theoretical equations, or through the analysis of real sounds, we seek a description of the impulse response of the system, as a sum of sinusoidal functions exponentially damped. Thus, the amplitudes, frequencies and damping factors of this impulse response will be directly related to F1, F2 and the delay \( \Delta \).

The transfer function of the system is given by
\[ T(\omega) = \frac{F1(\omega)e^{-\alpha_1 \omega \Delta}}{1 - F2(\omega)e^{-\alpha_2 \omega \Delta}} \]

then if one assumes that \( F(\omega) \) is real-valued, the power spectral density of \( T(\omega) \), \( S(\omega) \) is:
\[ S(\omega) = \frac{|F1(\omega)|^2}{1 - 2F2(\omega)\cos \phi + |F2(\omega)|^2} \]

The maxima of \( S(\omega) \) are obtained when \( \cos(\omega \Delta) = 1 \), that is for \( \alpha_n = \frac{\pi n}{L} \). In this case, the spectrum is harmonic, but its harmonicity can easily be taken into account by adapting the phase of \( F(\omega) \).

In order to approximate the behavior of these maxima, we suppose that the peaks are narrow enough to be considered as separated in the frequency domain. This assumption is generally reasonable for musical instruments.

Then, close to the resonance peaks we can note \( \omega = \omega_n \pm \varepsilon \ll 1 \) and \( \cos(\omega \Delta) \approx \cos(\varepsilon) \).

By using a limited expansion of \( \cos(\varepsilon) \) around \( \varepsilon = 0 \), one can approximate \( S(\omega) \) around each resonance:
\[ S(\omega_n) = \frac{F1(\omega_n)^2}{1 - 2F2(\omega_n)\cos \phi + F2(\omega_n)^2} \]

On the other hand, if one considers an exponentially damped sinusoid
\[ s(t) = C_n e^{\alpha_n t} \sin(\omega_n t + \phi) \]

its power spectral density is given by
\[ S(\omega) = \frac{C_n^2}{\omega_n^2 + \omega^2} \]

Similarly, by considering the local behaviour of \( S(\omega) \) around the resonance frequency (\( \omega = \omega_n + \varepsilon \)), we get:
\[ S(\omega_n) = \frac{C_n^2}{\omega_n^2 + \varepsilon^2} \]

By comparing (1) and (2), we find an expression of \( F(\omega) \) depending on \( \alpha_n \) and \( \varepsilon \):
\[ F(\omega_n) = \frac{2\varepsilon e^{\alpha_n \varepsilon} \left( \alpha_n^2 \varepsilon^2 + 2 \right) - 4}{2} \]

The filter \( F \) in the model takes into account the energy variations between the components, and is directly related to the constant \( C_n \) of the expression (2):
\[ F(\omega_n) = C_n e^{\alpha_n \varepsilon} \frac{F(\omega_n)}{C_n} \]
IV - Parameters estimation through the analysis of real sounds.

In this section we describe experiments, and the analysis method. The results obtained are compared with the theoretical values.

IV.1 - Experiments

The impulse response of the flute has been measured at an anechoic room by rapidly closing a finger hole, without exciting the flute with a jet stream at the embouchure. We have also considered guitar sounds, obtained under normal playing conditions. Actually, if we assume that the linear model is valid, the excitation performed on the string is not important for the parameters inside the loop, since it has an influence only on the filter F1.

IV.2 - Analysis Method

In order to compute the filters allowing the resynthesis of a real sound by the use of the waveguide model, we need to estimate the amplitudes, frequencies and damping factors of each partial. For that purpose, we use a spectral lines estimation technique based on time-frequency representations [6]. This method, based on the construction of an automatically matched bank of wavelet packets let us extract accurately the frequencies and amplitude modulation laws, even in the case of highly damped components, such as the ones we get in the flute case.

IV.3 - Results

IV.3.1 - Flute case

As seen in section III, the filter F2(n) depends on the delay d. In the flute case, the figure 2 shows the theoretical filter F2(n) for L, L/2, L/4 and L/6, where L=0.55m.

![Fig. 2 F2(n) for different tube lengths, L.](image)

The relation between the filter and the length (when neglecting the inharmonicity in the tube) is found to be

L \rightarrow \frac{L}{n} \text{ implies } F(n) \rightarrow F\left(\frac{n}{L}\right)

As discussed in section II, the value of \( \mu_n \) in the movement equation of a tube, depends on the friction near the body of the instrument, and is determined by comparing the real and the theoretical damping factors as seen in Fig. 3.

![Fig. 3 real( ) and theoretical( ) \( \mu_n \).](image)

From the experience, the estimated \( \mu_n \) is of order 6*10^-5. This value is much higher than the theoretical value in free field, which shows that the friction of the air near the body of the instrument can be seen as responsible of all the losses.

Figure 4 displays respectively the theoretical and estimated inharmonicity of the modes. The estimated inharmonicity is more important than the theoretical one. This may come from the way we produce the "impulse response" or from the previous estimation of \( \mu_n \).

![Fig. 4 Frequency( ) and fundamental( ) of a tube of length L=0.38m in the theoretical and real cases.](image)
IV.3.2 - Guitar case

Fig. 5 displays respectively the theoretical and estimated inharmonicity of a steel guitar string. The estimated data are in good accordance with the model.

Fig. 5 Frequency (in 1st harmonic) of a steel guitar string in the theoretical and real case.

Figs 6 and 7 display the respective Fourier transforms of the two estimated filters F1 and F2 corresponding to a guitar sound. Since the filters have been estimated directly on the sound itself, F1 includes the frequency response of the soundboard mechanically coupled with the string, and its radiative characteristics. Though, from a theoretical point of view, the filter F2 should be constant, one can observe greater losses for the high frequencies than for the lower ones. This is consistent with the fact that higher harmonics decrease faster than lower harmonics. This phenomenon can be justified theoretically by a loss coefficient R being a function of the frequency.

Fig. 6 The Fourier transform of the filter F1 corresponding to a guitar sound.

Fig. 7 The Fourier transform of the filter F2 corresponding to a guitar sound.

V - Conclusion

In this paper, we have presented a technique allowing the estimation of relevant parameters for the synthesis of real sounds by the use of a waveguide model, and compared the estimated data with their theoretical values deduced from simple propagative equations. Though we have considered two very different musical instruments, the flute and the guitar, we have seen that they can be processed in the same way, thanks to the similarity of the movement equations of longitudinal waves in fluids and transversal waves in solid beams. In the flute case, we have worked with the response of the air column excited with a transient, and the sound produced this way has been resynthesized faithfully. However, for a realistic simulation, the source problem (vortex phenomena, turbulence noise, non-linearities) together with the connection between the source and the resonator (the existing field in the tube interacts with the source) still remains.

In the guitar case, though, the sounds resynthesized this way are of very good quality, some problems still remain, namely the coupling between the string and the soundboard which is hard to modelize theoretically, and the radiation of the soundboard itself. Nevertheless, we believe that the first effect can be taken into account in the movement equations by the use of a loss term, being a function of the frequency. In order to get around the second problem, we shall make measurements of the string vibrations by the use of a laser vibrometer.

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References: