ESSENCE AS A MODALITY

A PROOF-THEORETIC AND NOMINALIST ANALYSIS

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Abstract

INQUIRY INTO THE METAPHYSICS OF ESSENCE tends to be pursued in a realist and model-theoretic spirit, in the sense that metaphysical vocabulary is *used* in a metalanguage to model truth conditions for the object-language use of essentialist vocabulary. This essay adapts recent developments in proof-theoretic semantics to provide a nominalist analysis for a variety of essentialist vocabularies. A metalanguage employing explanatory inferences is used to individuate introduction and elimination rules for atomic sentences. The object-language assertions of sentences concerning essences are then interpreted as devices for marking off structural features of the explanatory inferences that, under a given interpretation, constitute the contents of the atoms of the language. On this proposal, object-language essentialist vocabulary is *mentioned* in a proof-theoretic metalanguage that *uses* a vocabulary of explanation. The result is a nominalist interpretation of essence as a modality, understood in the grammatical sense as a modification of the copula, and a view of metaphysical inquiry that is closely connected to the explanatory commitments present in first-order inquiry into things like sets, chemicals, and organisms. This result illustrates that some of the presuppositions that have animated analytic metaphysics over the last few decades can be profitably substituted with more practice-oriented conceptions of the forms of reasoning at work in different domains of human knowledge.¹

1. A Metalinguistic Approach to the Metaphysics of Essence

1.1 Introduction

Fine (1994) points out that the following two sentences share a truth-value at all and the same possible worlds:

Socrates exists

{Socrates} exists

1. The current essay forms a triad with Stovall (2019) and Stovall (2020) in providing a more solid formal footing to some of the material in the first two chapters of Stovall (2015). I owe separate debts of gratitude to Robert Brandom and Nissim Francez for helping me to think in the ways I try to articulate here. Work on this essay was supported by the joint Lead-Agency research grant between the Austrian Science Foundation (FWF) and the Czech Science Foundation (GAČR), *Inferentialism and Collective Intentionality*, GF17-33808L.

As a consequence, Fine argues, the modalities of metaphysical necessity and possibility cannot supply truth conditions sufficient to make the following true:²

Socrates is essential to {Socrates}

and the following false:

{Socrates} is essential to Socrates

In Fine (1995a), similar remarks are made about the inability of metaphysical modality to account for the asymmetric ontological dependence of {Socrates} on Socrates. This is a problem because it seems a set is the kind of thing whose existence *depends upon* the existence of its member(s), or which has its members as (part of) its *essence*, rather than *vice versa*. Fine argues that the way out of this difficulty is to see that talk of essence involves more than talk of metaphysical modality. He does this by adopting a realist metalanguage that speaks of the identities or natures of objects, and of what is true in virtue of those identities (1994, p.9):

... different essentially induced truths may have their source in the identities of different objects.... In particular, an induced truth which concerns various objects may have its source in the nature of some of these objects but not of others. This is how it is with our standard example of Socrates being a member of singleton Socrates; for this is true in virtue of the identity of singleton Socrates, but not of the identity of Socrates.

Instead of reducing essence to modality, in that essay Fine suggests we reduce metaphysical modality to essence: The metaphysically necessary is that which holds in virtue of the identities of all objects whatsoever (1994, p.9). Fine's (1995c) and (2000) supply a logic and semantics for essentialist vocabulary, and in more recent work (2012, 2015), he has turned to the notion of metaphysical

2. Metaphysical statements permit a wide range of linguistic forms – e.g., we could speak of truth-making between sentences, or between propositions, or we could speak of dependence relations among facts, or among objects and their properties and relations. I wish to be neutral on these issues and so, with the exception of the lightly regimented essentialist vocabulary introduced later, I do not presume that one way of speaking should be emphasized over another.

grounding and its relationship to essence.

While Fine's views have developed (cf. the discussion in the opening pages of his 2015), the approach toward essence he pioneered has done much to shape the field over the last two and a half decades. There are those who, contra Fine, broadly favor the reduction of essence to modality (Correia 2007, Cowling 2013, Wildman 2013 and the last section of his 2016, and Zalta 2006), and there are those who are broadly critical of the reduction of essence to modality (Almog 2003, Gorman 2005, Morvarid 2017, Skiles 2015, and Wildman 2016). Conversely, there are those who are broadly in favor of the reduction of modality to essence (Correia 2006 and 2012, Fine 1994 and Fine 1995b, and Hale 1996) and those who are broadly critical of the reduction of modality to essence (Teitel 2019, Wildman 2018, and Zylstra 2019). And some have argued that both modality and essence should be taken as primitive notions (Hale 2013, Jubien 2009). Reductions of metaphysical grounding to essence have also been proposed (Correia 2013, Dasgupta 2014, Rosen 2010), and of essence to ground (Dasgupta 2014, Tillman 2016), as have reductions of essence and ground to a generalized notion of identity (Correia and Skiles 2017), though criticisms of these reductions have also been advanced (Carnino 2014, Fine 2012 and 2015, and Nutting et al. 2017). Meanwhile, philosophers are beginning to examine the connection between essence and explanation (Christensen 2014, Dasgupta 2014, Glazier 2017, Tillman 2016; Koslicki 2012 argues that this examination was already underway in Aristotle), sometimes by way of (or alongside) the notion of metaphysical grounding. This development stands out in Fine's recent discussions of essence (e.g., 2012 and 2015).

In this paper, I put pressure on some of the model-theoretic and realist presuppositions that have shaped these debates, and I offer a proof-theoretic and nominalist conception of a variety of essentialist vocabularies. The view is nominalist in the sense that it treats talk of essence through the categories of grammatical rather than metaphysical modality: To speak in an essentialist register is to modify the copula by placing linguistic expressions into a space of reasoning governed by explanatory inferences. By embedding a view of this sort into an appropriately structured proof-theoretic semantics, object-language talk of essence can be understood in terms of our commitments to explanatory inferences that are good at every context. In this way, talk of essence goes

proxy for talk of universal explanation: Essentialist talk enables interlocutors to communicate about the explanatory structure of a shared language without having to adopt the explicitly metalinguistic resource of the explanatory inference. To forecast, the appearance of an asymmetric essential relationship between Socrates and {Socrates} is accounted for by the fact that we explain the existence of sets on the basis of the existence of their members, but not *vice versa*. And this explanation is derived from reflection on the practice of identifying and individuating sets and human beings. For we establish that two sets are identical by showing that they include all and the same members (to go metalinguistic: We list the names of the members of each set, or otherwise denote them as groups, and verify that exactly the same names (groups) occur on both lists). But we do not identify and individuate human beings in terms of the sets containing them.

It will take time to appreciate the details of the proof-theorist's perspective, and the details are unavoidably technical. But the result is a framework for something deserving the title "metaphysical research" that is an effort to reconstruct the logic of the various sciences. For the resources of proof theory underwrite a nominalism whose development proceeds not on the basis of speculative metaphysical models, but from a reflection on the capacity for, or on the conditions under which we exercise, ordinary world-regarding cognition. Given the degree of sophistication proof-theoretic semantics acquired over roughly the same period of the development of model theory, it will take some work to exhibit just how to make good on this nominalism.

At the outset, it may help to frame the general view in terms of its place in the field (impatient readers can skip to the closing paragraph of section 1.3 and proceed from there). Despite centuries of more-or-less continuous philosophical research into theories of meaning, there remain deep and unresolved questions about the nature of cognition as having both extension in what a thought is *about*, *represents*, or *refers to*, and comprehension in *what is said* about the things we talk about. A distinction between extension and comprehension in cognition or linguistic meaning has been marked in a number of ways in the history of philosophy, often in the context of an observation that substitution of co-referential terms fails to preserve truth value in modal contexts (see chapter 1 of Stovall 2015 for an overview). In general, extensional notions of meaning

are founded on relations between the world and our language or concepts, while comprehensional notions of meaning are spelled out in terms of intralinguistic or concept-containment relations. This distinction has been drawn with different terms by various figures – "*suppositio*" and "*significatio*" by the scholastics, "*étendue*" and "*compréhension*" in the logic of Port-Royal, "*extension*" and "*intension*" by Leibniz, "*Umfang*" and "*Inhalt*" by Kant, "denotation" and "connotation" by Mill, "scope" and "force" by DeMorgan, "breadth" and "depth" by William Hamilton and the early Peirce, "*Bedeutung*" and "*Sinn*" by Frege, "extension" and "intension" by Carnap, and "denotation" and "sense" by Church. Tangled lines of thought run throughout this history. Nearly bookending the period of development just sketched, whereas the medievals used "*suppositio formalis*" and "*suppositio materialis*" to denote use and mention contexts respectively, "formal mode" in Carnap is the explicitly metalinguistic idiom that has its "material mode" counterpart in the use of object-language vocabulary. And our own use of the term "intension", denoting a function from a world to an extension, is largely a product of Carnap's decision to use that term in replacing Frege's notion of *Sinn* with something more formally tractable (cf. Carnap 1947 and the discussion in Peregrin 2007). Carnap's models employed state descriptions rather than possible worlds, however, and while some of the ideas that Kripke drew together were already extant in the literature (see the discussion at section 3.2 of Ballarín 2017), with the advent of Kripkean semantics philosophers and logicians gradually took themselves to be permitted to use alethic modal notions with confidence (but see Tuboly 2015 and 2018 for an account of the development of modal logic during this period that goes against the conventional narrative).

That confidence was not groundless, of course, as Kripke's use of possible worlds not only established soundness and completeness results for the logic of necessity and possibility; he also demonstrated that features of the axiom systems of C.I. Lewis were recoverable in terms of the mathematical properties of the accessibility relation between possible worlds (Kripke 1963 and 1965, and see Sedlar 2009 for an argument that C.I. Lewis' view was closer to Kripke's than is commonly supposed). This in turn allowed for a formal specification of whether something could be, e.g., *necessary* but not *necessarily necessary*, denial of which amounts to accepting the characteristic axiom of S4: It all

depends on whether the accessibility relation is transitive. On the face of it, this seems like a real advance in understanding the way terms like “necessarily” function. It comes at the cost of canalizing our thought about modality in terms of word-world relations of representation, but that cost was worth bearing for many. As a consequence, we risk losing the idea that modal contexts – and intensional contexts more generally – can be understood in terms of a notion of meaning involving word-word or concept-containment relations.

Owing to the successful applications of possible-world semantics across a wide range of topics in Anglophone philosophy and linguistics over the last half century, “possible-world semantics” and “intentional semantics” have become co-extensional in many circles (Braun 2012 surveys post-Fregean technical developments in semantics and philosophical logic while carefully avoiding this assimilation). Carnap himself is clear that he breaks from historical tradition in his use of the term (1947, §11), however, and for his own purposes this posed no problem. He aimed to lay down a semantics for the alethic modal operators in terms of their truth conditions, and the device of intensions as functions from state descriptions to extensions suits that task. Furthermore, his decision led to fruitful research programs in philosophy and related areas, as the subsequent history of intensional semantics has shown. But the proliferation of interest in so-called “hyperintensional” semantics, ushering in what Nolan (2014) has called the “hyperintensional revolution” in metaphysics, can be understood both as an artifact of Carnap’s terminology here, and as evidence that (as Carnap himself understood) the notion of intension he introduced was not adequate as an *Ersatz* for the notion of concept-containment that philosophers had been developing for centuries. One of the aims of the current essay is to provide a proof-of-concept for the viability of a metaphysical research program that, instead of attempting to reconstruct a notion of intensional meaning *inside* model theory, uses the resources of proof theory to provide a complement to model-theoretic notions of meaning.

I offer a few remarks on this larger program at the end of the essay, and the intervening pages focus on that proof-of-concept. The result is 1) a plausible view about the proof-theoretic semantic function of a variety of essentialist vocabularies habitually employed by philosophers and in natural language English, and 2) an argument that, to the extent the notion of essence is philosophically

interesting outside of purely formal work, it can be developed by attending to the explanatory practices of the various sciences. Section 1.2 outlines the essay’s problem space while section 1.3 surveys the areas of research it speaks to, and closes with an overview of the rest of the essay.

1.2 *A Metalinguistic and Nominalist Alternative to Metaphysical Realism About Essence*

Though they differ in the metaphysical commitments they’re willing to take on, the philosophers cited in the first two paragraphs of section 1.1 share a commitment to realist interpretations for talk of essence, ground, etc. In each case, metaphysical vocabulary is used in a metalanguage to interpret the metaphysical vocabulary appearing in the object language. Fine variously says that essentialist claims are true in virtue of the identity of certain objects (1994, 1995c, 1995b) or true in virtue of the nature of those objects (Fine 2000); Correia (2006) defines metaphysical necessity in terms of what is true in virtue of what it is to be some way, and in (2012) he glosses essence as basic nature and uses this notion to distinguish metaphysical from conceptual and logical truths; Cowling (2013) defines the nature of an object in terms of its existence at different worlds and the possession of certain properties; Nutting et al. (2017) define the essence of an object in terms of pluralities of properties; Rosen (2010) defines grounding in terms of relations between facts that obtain in virtue of what those facts are, while arguing that some facts are grounded in the essences of objects; and Correia and Skiles (2017) give a unified account of essence and grounding in terms of a generalized notion of identity. Investigations of this sort are commonly characterized as distinctively metaphysical and contrasted with the kinds of investigations that arrive at conclusions drawn in mathematics or the natural and social sciences (cf. Fine 2012, pp.37-40, and 2015, p.296 and pp.310-11). Some philosophers are apprehensive about metaphysical notions and would prefer to do without them, however – worries are sometimes raised in print that the notions employed are “crazy” or “too obscure” to be relied on (Correia 2012, p.641), and in my experience these concerns are not uncommon in conversation about projects in analytic metaphysics. But intuitions about the asymmetric ontological dependence of a set on its members are widespread among contemporary philosophers, and either way, we habitually employ essentialist vocabulary in

everyday speech (e.g., “time is of the essence”).

The influence of model-theoretic metalanguages on this field has led to the rebirth of a sort of metaphysical speculation that, from the standpoint of recent advances in proof-theoretic semantics, is optional. For it is open to the philosopher to interpret essentialist vocabulary nominalistically, by employing a metalanguage of rules within a derivation system, similar to proposals that interpret the apparently property-denoting sentence:

red is a property

in terms of the explicitly linguistic sentence:

“red” is a one-place predicate³

The proposal is to interpret claims about essence as expressions of commitment to context-invariant explanations in a derivation system governed by explanatory rules of inference. Talk of essence can thus be understood as implicit talk of a context-invariant explanatory inference ticket. And so object-language talk of essential relations goes over into metalinguistic talk concerning which explanations are warranted in a proof system, and because essentialist vocabulary is *mentioned* in the proof-theoretic metalanguage without being *used* there, its object-language use can be understood as a means for giving expression to these explanation-backed rules of inference. In this regard, a putatively *ontic* use of essentialist vocabulary is rendered in *deontic* terms.

1.3 Target Audience

The results of this essay should be of principal interest to three groups of researchers: 1) Metaphysicians working on notions of essence and fundamentality;

3. This is an old idea, going back at least to medieval theories of material supposition (e.g. in Ockham). In our own time, the contention that what appears as fact-stating metaphysical or “material mode” discourse is actually rule-based discourse concerning the use of language is most closely associated with Carnap (2002), first published in German in 1934 and appearing in English in 1937. Bar-Hillel (1963) regards this understanding of metaphysical disputation as the “greatest single achievement of Logical Empiricism (and of Analytical Philosophy in general)” (pp.533-37). For the shift in Carnap’s perspective on metaphysics that led to this breakthrough, see Awodey and Carus (2007). For recent treatments of this broadly Carnapian point, see Thomasson (2016) and Hofweber (2009); for a more Gibbardian approach, see the ontological expressivism of Flocke (Forthcoming).

2) philosophers interested in the interface between philosophy and science – particularly concerning questions of essence and fundamentality – but who may distrust metaphysical speculation of the sort that has developed in analytic metaphysics over the last few decades; and 3) formally minded philosophers interested in the expressive strengths of different metalanguages, and the use of proof theory as a basis for developing a nontrivial, formally tractable, and scientifically minded approach toward apparently metaphysical inquiry.

As concerns metaphysicians, a proof-theoretic and nominalist analysis of essence provides a basis for reconceiving metaphysical inquiry as an engineering problem in the logic of the sciences. Because the model-theoretic realist interprets object-language essentialist vocabulary in terms of the use of metalinguistic metaphysical vocabulary, she must determine which metaphysical relations to accept in her metalanguage in order to interpret the semantic values of the corresponding object-language sentences. The proof-theoretic nominalist, on the other hand, can interpret object-language essentialist talk in terms of the explanations given in different domains of inquiry. As a consequence, ordinary practices of explanation suffice to determine which essentialist claims to go in for. In each case, I shall argue, the explanations at issue appeal only to prosaic features of the way we go about identifying and individuating sets, chemicals, human beings, etc. It turns out that commitments to a metaphysics of essence do not arise from the semantics needed to make sense of the way we explain, e.g., that some human being is identical with a particular zygote, or that a set is constituted by its members. And so intuition-driven model-theoretic metaphysical speculation is replaced with the task of developing a proof-theoretic examination or reconstruction of the explanations given in different domains of inquiry.

This essay will also be of interest to philosophers, particularly philosophers of science, who may harbor suspicions about the merits of analytic metaphysics. I mean to include researchers interested in scientific essentialism (e.g., Bird 2007, and Ellis 2001 and 2002), as well as those working on the foundations of science and scientific explanation (book-length treatments from the last two decades include Friedman 2001, Ladyman et al. 2007, Lange 2009 and 2016, Lipton 2004, Norton, n.d. Strevens 2008, Wilson 2006, and Woodward 2003). Because the proof theorist looks to pre-existing practices of explanation

in order to semantically evaluate different object-language essentialist claims, philosophical inquiry into the metaphysics of essence must proceed by way of an inquiry into the explanatory practices that occur in the sciences. We should not expect to settle whether to accept that something is part of the essence of something else except insofar as we can settle whether the corresponding context-invariant explanation holds; and to settle that latter issue, one must look to the explanations that are employed in the domain of inquiry in question. This is not of itself to deny the propriety of prescriptive metaphysics, but it is to say that the task of *descriptive* metaphysics must be well on its way before prescription can even seem sensible (the distinction between prescriptive and descriptive metaphysics is owed to Strawson 1959).

Finally, the details of this analysis are meant to contribute to ongoing work in proof-theoretic semantics, inferentialism as a theory of linguistic meaning, and areas of philosophy influenced by this approach in philosophical logic. This essay contributes to a growing literature that draws on proof theory as a basis for investigating areas of philosophy like metaphysics and scientific explanation that have traditionally been conceived model-theoretically (work like Khalifa et al. 2018, Litland 2015, 2016, 2017, and 2018, Millson et al. 2018, Millson and Straßer 2019, and Poggiolesi 2016 and 2018 stand out in this regard). Along with the proof-theoretic analysis of generic sentences and atomic sentences in Stovall (2019) and (2020), this essay helps to establish the relevance of proof theory as a basis for re-examining the model-theoretic presuppositions that have shaped some of the core areas in philosophy over the last few decades.

The rest of the essay is structured as follows. Part 2 characterizes essentialist vocabulary as modal vocabulary. Part 3 introduces the proof-theoretic semantics of Francez (2015) and extends that semantics by specifying introduction and elimination rules for atomic sentences, under an intended interpretation, in terms of explanatory inferences. Those familiar with inferential expressivism and its history may glance over part 2, and those familiar with proof-theoretic semantics may glance over section 3.1; sections 3.2 and 3.3 should not be skipped. Part 4 is the core of the paper. There, I use this semantics to provide nominalist interpretations for a variety of notions of essence that have been discussed in the philosophical literature. In the process, I show how to use first-order inquiry as a basis for specifying semantic evaluations for different uses of essentialist

vocabulary, and I argue that such talk should not be seen as the contentious sort of metaphysical speculation it is sometimes treated as. Part 5 then summarizes the discussion, canvasses some open questions that remain to be answered, and indicates how a more general proof-theoretic nominalist metaphysics might proceed.

2. Background

2.1 *Modality as Semantic Content and Modality as Grammatical Mode*

Though there are different ways of lexically representing essentialist claims (e.g., with a sentential operator or a predicate modifier), it is generally accepted that systematic translations of one form can be given in terms of another. I will interpret essence as a grammatical modality, and so my official view is that talk of essence is to employ the adverb “essentially” as a predicate modifier. Because at times this is awkward, I will occasionally employ other formulations; translations into adverbial form can be substituted as required.⁴

To treat essence as a modality in a grammatical sense is to go against standard philosophical use of the term “modality”, on which modality concerns what is necessary, possible, and impossible. This trend is so ingrained that arguments for and against a reduction of essence to metaphysical necessity are often characterized as arguments for and against a modal analysis of essence (e.g. in the title of Fine’s 1994, “Essence and Modality”). Linguists working on natural language semantics distinguish different “flavors” of modality, however, and the alethic modalities of necessity and possibility are only one among them – there are the tense modalities for past, present, and future states; the epistemic modalities concerning what must, might, and cannot be permitted as additions to a belief state; the bouletic modalities concerning what is desired and disliked; the normative and deontic modalities concerning how agents, artifacts,

4. In the literature on grounding, one position holds that grounding should be understood as an operator on sentences rather than a relation between, e.g., objects or facts (cf. Fine 2012, §1.4). Insofar as talk of such a relation involves more substantive ontological commitments than does use of an operator on sentences, a nominalistically inclined philosopher might adopt a similar approach for talk of essence. However, if the metalanguage that models truth conditions for sentences governed by this operator remains realist, in the sense that metaphysical language is used in the metalanguage, then there will remain ontological commitments that the current essay aims to avoid. My thanks to a reviewer for suggesting I say something about this possibility.

organisms, et al. ought and ought not be and behave, etc. (cf. Kratzer 1977 and von Fintel 2006). Rather than restricting “modal” to cover only talk of necessity and possibility, I propose to treat essence as a modality in the linguistic sense that it functions to *modify* the copula.⁵

There is both etymological and philosophical support for this move. The term “essence” is a nominalization of the Latin copula “*esse*”, used by Latin translators for the Greek “*to ti ên einai*” (“what it is/was to be”) – which Aristotle associates with the definition of a thing’s *ousia* (substance) – so as to discuss Aristotle’s metaphysics in Latin. To characterize some essentialist vocabulary as a grammatical modality is to think of that vocabulary as modifying the copula, analogous to the role performed by adverbs and adjectives in sentences like “the dish was horribly under-cooked” and “her filling the cistern was intentional”. In these sentences, the terms “horribly” and “intentional” do not describe a feature of the event or action that could be picked out as *the horrorability* or *the intentionality* in the same way that we can pick out the under-cooked dish and the filled cistern (it was for similar reasons that adverbial theories of sensation promised to account for sensory experience without inflating our ontology with sense-data). Instead, these terms operate to modify the relation between a subject and a verb, performing a role in orienting our thinking about some object or event by characterizing it in a certain way. Grasp of that characterization requires we know something about what it implies and rules out – e.g., that the dish wasn’t palatable, and perhaps, in context, that the oven still needs repair.

The suggestion that essentialist vocabulary modifies the copula is reinforced by features of natural language, where talk of essence often takes the adjectival and adverbial forms. In conversation, these devices function as a means for shortening an explanation by way of marking certain features as relevant (in some sense) for understanding some situation:

When the only good quarterback on a team without a running

5. Wiggins (1976) and Peacocke (1976) provide an adverbial (copula-modifying) and *de re* reading of necessity so as to analyze talk of essence: To say that some property *P* is essential to some object *o* is to say that *o is-necessarily P*. Their proposal differs from mine in two significant respects: First, theirs is a notion of essence as a modality understood in terms of necessity rather than grammatically; second, their truth-conditional analysis of *de re* necessity is realist. My thanks to a reviewer for bringing these essays to my attention.

game gets hurt, the game is essentially over.

A charged battery is essential to a running car.

He was, essentially, being belligerent all morning.

In this company, a good work ethic is essential to success.

Or consider a remark from a philosopher none too smitten with a metaphysics of essence:

Existence statements in this philosophical vein do admit of evidence, in the sense that we can have reasons, and essentially scientific reasons, for including numbers or classes or the like in the range of values of our variables. (Quine 1969, p.97)

The assertion of such sentences directs an interlocutor’s attention toward aspects of a situation the cognitive navigation of which is, in context, instrumental for understanding what is going on and/or cooperating with one’s interlocutors (notice that “essential” would work for “instrumental” in the current sentence). At least some uses of the noun “essence” can be thought of as reason-guiding as well. When someone says “time is of the essence”, the intended effect is to induce the auditor to make decisions that will shorten the length of time it takes to finish his or her activities; there is no suspicion that a subtle view in the metaphysics of time is in the offing.

Of course the philosopher’s sense of “essence”, owing its existence to Aristotle’s metaphysics, will involve greater context-invariant explanatory relations than does talk of the essentiality of a strong work ethic for professional success. But at this point, none of these claims need be thought to report some metaphysically deep or interesting feature of the world. Instead, they can be read as guideposts for directing auditors around the relations of explanatory consequence that different sentences stand in at these (and similar) contexts of utterance, while ordinary facts about the situations in question – e.g., about the importance of a strong work ethic for professional success – will explain why these relations hold.

Finally, to treat essentialist language as a mode of relating representations to one another in a proof system, and thus as disguised metalinguistic talk for

guiding an auditor around the space of reasoning that governs the ordinary representational sentences of a language, is to underscore a point made by Kant in his discussion under the “modality” heading of the logical function of the understanding in judgment. Whereas the functions falling under the headings of “quantity”, “quality”, and “relation” all contribute to the content of a judgment, Kant writes (emphasis added):

The modality of judgments is a quite special function of them, which is distinctive in that it contributes nothing to the content of the judgment (for besides quantity, quality, and relation there is nothing more that constitutes the content of a judgment), *but rather concerns only the value of the copula in relation to thinking in general.* (*Critique of Pure Reason*, A74/B99-100)

Grammatically and philosophically, then, there are (*pro tanto*) good reasons for treating “essence” as a nominalization of the copula and thereby understanding the use of essentialist vocabulary as a mode of relating terms and sentences to one another in a space of reasoning, rather than as part of the representational content of a judgment.

2.2 Expressivism and Representationalism

There is a natural inclination to interpret declarative sentences as representations of the way things are. When this inclination is systematized as a theory of meaning, the result is a picture on which sentences mean what they do in virtue of the way they represent the world (Belnap 1990 refers to this as the “declarative fallacy”). Given that the nominalist analysis for “red is a property” is likewise in the declarative (“‘red’ is a one-place predicate”), this view of meaning would commit the nominalist to supposing that the sentence “red is a property” represents facts about language. The assimilation of declarative grammar to representational semantics should be resisted by the nominalist, however, as can be readily seen by considering the following counterfactual: Had there been no language-users, there would have been no predicates, yet the color red would still have existed. What this shows is that one who says “red is a property” is not *saying* or *representing* that “red” is a one-place predicate.

Instead, the nominalist should interpret the use of metaphysical vocabulary

non-representationally. And it is a feature of natural language that not every sentence in the declarative mood need be construed as a representation of the way things are. Said from one friend to another, a sentence like “the cups go in the pantry” may be a description of the way the house is organized, but it may also be a request or a suggestion. And said from an employer to an employee, it may be a command. At other times, a declarative sentence might be used to triangulate agency in the shared pursuit of an end. When packing for a move, an utterance of “the cups would break if they aren’t wrapped in packing paper” functions to guide the decisions of the auditors with regard to how to handle the cups. Similarly, the nominalist about metaphysical declaratives may be read as offering an expressivist theory of those sentences: In asserting that red is a property, I express commitment to the claim that “red” is a one-place predicate, though I do not say or represent it.⁶ This position is compatible with a minimalist view about the representational features of sentences in the declarative mood, allowing that the sentence “red is a property” *represents that* red is a property, *is true if and only if* red is a property, etc. But these minimal commitments concerning when the truth predicate and related notions are apt need not commit one to a theory of meaning that uses metaphysical language in its metalanguage.

3. Proof Theory as a Basis for Expressivist Semantics

3.1 Overview of Proof-Theoretic Semantics

In this section, I introduce the semantics developed in Francez (2015), and in the rest of part 3 I extend it to provide an analysis of the semantic values of atomic sentences.⁷ In what follows, I assume familiarity with natural deduction

6. It is a hallmark of post-Carnapian expressivist interpretations of metaphysical vocabulary that one distinguishes the analysis of what an object-language sentence expresses about the metalanguage from an analysis of what it (purports to) say about the world when used in the object language; this distinction is drawn by Sellars in §§78 and 81 of (1958); by Thomasson in §4 of (2009); by Price in §5.1 of chapter 2 of (2013); and by Brandom in chapter 5 of (2015).

7. There is more to proof-theoretic semantics and its historical roots than I can give space to here, but three areas of research are particularly central to the program: Philosophical logic and formal semantics, metaphysics and philosophy of language, and natural language semantics. Francez (2015) is a magisterial treatment of the first and the third of these areas. The formal basis for proof theory was laid with investigation into sequent calculi and natural deduction systems by Gerhard Gentzen (1935a and 1935b) and Dag Prawitz (1965, 1971, and 1973). A representative sample of more recent work on this front is found in Schroeder-Heister (1984) and (2008), Hallnäs (2006), Hallnäs and

systems for propositional logic; an overview of such systems can be found in chapter 2 of Francez (2015).

Consider the class of rule-governed natural deduction systems **ND** (classical, intuitionistic, etc.), each of which is defined over a language $\mathcal{L}$ consisting of a denumerable set of atomic sentences closed under composition from the Boolean operators in the usual way. The rules of each natural deduction system $ND_i \in \mathbf{ND}$ induce an interpretation of the sentences of $\mathcal{L}$ by defining the class of derivations permitted in $\mathcal{L}$. For my purposes, it does not matter whether the natural deduction system is classical, intuitionistic, or otherwise, and I leave reference to a particular interpretation-inducing ND_i implicit.

The following conventions are used. Lowercase Greek letters name logically complex sentences, lowercase English letters p and q name atomic sentences, and uppercase Greek letters name sets of sentences; all of these may be subscripted. In part 4, I consider subsentential structure for the atoms of the language, where lowercase English letters s and t name singular terms and uppercase English letters name predicates. Strictly speaking, these letters are schematic letters in the metalanguage, but I use them both schematically and as instances throughout.

In order to determine the proof-theoretic semantic value of a logically complex sentence, one must specify the rules that govern its occurrences in the conclusions and/or premises of derivations. There are correspondingly two sorts of rules of inference for each logical operator: The introduction rules specify when an operator can be introduced as the main operator in the conclusion of an inference, and the elimination rules specify when it can be eliminated in a premise to draw some other conclusion. The introduction rule for conjunction, for instance, fixes the conditions under which one can infer *to* a sentence with a conjunction as the main operator:⁸

Schroeder-Heister (1990) and (1991), Martin-Löf (1996), Prawitz (2006), and Francez (2014), (2015), and (2016). Core works developing proof-theoretic approaches to metaphysics and the philosophy of language include Carnap (2002), Sellars (1958) and (2007), Dummett (1993), and Brandom (1994) and (2008). Finally, proof theory has been used as a basis for natural language semantics in Francez and Dyckhoff (2010), Francez et al. (2019), Więckowski (2011), part II of Francez (2015), and Stovall (2019) and (2020).

8. The horizontal line represents a relation of proof or inference. The sentences above the line are the premises, the sentence below is the conclusion, and the notation to the right indicates the rule that entitles the inference.

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$$\frac{\phi \quad \psi}{\phi \& \psi} \&I$$

while its left and right elimination rules determine what follows *from* a sentence governed by that operator:

$$\frac{\phi \& \psi}{\phi} \&EL \quad \frac{\phi \& \psi}{\psi} \&ER$$

A derivation is a series of inferences each of which is justified by a rule of inference. The introduction and elimination rules for the logical operators determine a set of *canonical derivations* for every logically complex sentence. Those derivations are used to define the semantic values of these sentences.⁹ A derivation *to* some logically complex sentence ϕ is I-canonical just in case it ends in an inference to ϕ whose final step is justified by an introduction rule for the major operator of ϕ .¹⁰ A derivation *from* some logically complex sentence ϕ is E-canonical just in case the first rule applied to ϕ as a major premise in the derivation is an elimination rule for the major operator in ϕ .¹¹ These two classes of canonical derivation determine two corresponding sorts of proof-theoretic semantic content or value. Whereas model theories lay down extensions or denotations for an interpreted language, the rules of proof theory specify comprehensions or connotations as follows:

DEFINITION 1 (*I-Canonical and E-Canonical Comprehension*)

the *I-canonical comprehension* of ϕ at $\Gamma =_{def}$

the collection of all I-canonical derivations of ϕ from Γ

the *E-canonical comprehension* of ϕ at $\Gamma =_{def}$

the collection of all E-canonical derivations from ϕ at Γ

9. This idea is due to work by Prawitz concerning the meaning of sentences in formal languages, e.g., his (1973). See Francez (2016) for an overview.

10. Francez (2015) and (2016) impose an additional condition in order to have the commutativity and associativity of disjunction count as part of its meaning. As that detail does not bear on what follows, I suppress it here.

11. This is a more general definition than is provided in Francez (2015), p.36, which defines E-canonicity in terms of derivations that *start* with an application of an elimination rule for the relevant sentence. The definition here allows that a derivation containing inferences before the application of an elimination rule for a sentence ϕ may still count as part of ϕ 's meaning (my thanks to Nissim Francez for discussion on this issue).

The reference to Γ allows for derivations from open assumptions to count as meaning conferring. From these definitions, it follows that proof-theoretic semantics is hyperintensional insofar as the proof theorist can distinguish the semantic content of ϕ and $\phi \& \phi$ – for the derivations to and from these sentences differ.¹² Notice also that the compositionality of the language is accounted for in terms of the compositionality of the derivation relation induced by the rules of the proof theory (for more on this feature of proof-theoretic semantics, see Francez et al. 2019, chapter 5 of Francez 2015, and section 3.4 of Stovall 2019).

One need not suppose that I-canonical and E-canonical derivations jointly determine meaning, however, and one or the other might be given priority. Francez (2015), pp.41–42, distinguishes four positions. *Inferentialism* is the view that the semantic value of a logically complex sentence ϕ is the set of I-canonical derivations for ϕ , with the E-canonical derivations justified by reference to the I-canonical derivations. *Pragmatism* is the view that ϕ 's semantic value is the set of its E-canonical derivations. The *combined view* holds that both the I-canonical and the E-canonical derivations for ϕ jointly determine its semantic value. Finally, the *non-uniform view* takes a mixed approach to canonicity, holding that different operators are associated with different types of canonicity. The dominant trend in proof-theoretic semantics, stretching back to Gentzen's work in the 1930s and Prawitz's work in the 1960s, is to hold that the introduction rules are definitive of the operators they govern, with the elimination rules derived from them according to some principle. To put the point as Gentzen does (metaphorically, as both Gentzen and Prawitz note): "[A]n introduction rule gives, so to say, a definition of the constant in question", while "an elimination rule is only a consequence of the corresponding introduction rule" (quoted in Prawitz 1965, p.33, footnote). I will likewise adopt the inferentialist position and treat I-canonical derivations as determinative of a sentence's semantic value, though nothing in what follows turns on doing so. And for the purpose of generality, I discuss introduction and elimination rules separately.¹³

12. See the discussion in Pezlar (2018).

13. Though beyond the scope of the present essay, there are important consequences for how one conceives of semantics that are bound up in the moves the proof theorist makes here. See Hjortland and Standefer (2018) for an overview.

3.2 Proof-Theoretic Semantic Values for Atomic Sentences

Investigation into proof-theoretic interpretations for atomic sentences is a relatively recent development in proof-theoretic semantics, and semanticists tend to interpret the canonical derivations for atoms either 1) as the single-step inferences from the atom to the atom (as in Francez 2015), or 2) they are stipulated according to definitional systems of the sort found in Prawitz (1973), or 3) they are assumed given from outside the proof system (e.g., Prawitz 1973, p.231; Francez et al. 2019, p.383–84; Francez 2016, p.481; and Więckowski 2011, §2). Francez (2015), p.377, and Pezlar (2017) emphasize that the task of rendering a proof-theoretic interpretation of atoms has yet to be finished. Nevertheless, within the context of Francez (2015), the shape that the result will take is clear. Extending Francez's proposal from logically complex to atomic sentences, to specify an intended interpretation for an atomic sentence is to associate that sentence with sets of derivations to it and from it according to its introduction and elimination rules – Dummett (1981, pp.74–6 and 453–6) refers to these generalized introduction and elimination rules as the *circumstances* and *consequences* of application of a sentence in. Where the intended interpretation is English, the sentence "the match is lit" may have

the match is struck, oxygen is present
the match is lit

as an introduction rule or circumstance of application and

the match is lit, the match is dropped, the rug is soaked in oil
the rug is (or will be) on fire

as an elimination rule or consequence of application. What is needed is some way of determining, for a given interpretation, when one is entitled to make inferences of this sort.

The notion of an explanation has certain features that make it well-suited for this purpose. An explanation can be understood as a kind of inference, and just as there are both introduction and elimination rules, so are there two orders or directions of explanation. On one hand, we may keep a sentence fixed and look to see which contexts or circumstances better explain its truth. On the other hand, we may consider adding an interpreted sentence to different contexts and look to see which other interpreted sentences are better explained by it. Call the former

the *circumstantial order of explanation* and the latter the *consequential order of explanation*. In both cases, contexts can be understood as sets of sentences that, under an interpretation, entitle an explanatory inference to some conclusion. In the material mode, however, the interpreted sentences that constitute a context are traded for an understanding of a context as a situation that explains why some fact obtains. Because semantic descent of this sort is possible – from talk of interpreted sentences and explanatory inferences, to talk of facts and explanations – I make free use of a metaphysics of fact and explanation in what follows. As I emphasize in section 4.2, the use of essentialist vocabulary is still avoided. In the interest of readability, I likewise speak equivalently of both (circumstantial) explanations, and contexts for explanatory inferences

To appreciate the two orders of explanation, consider the following example. Suppose that Phaenarete and Sophroniscus, the parents of Socrates, met in a market. In general, their meeting in a market is a poor explanation for the existence of Socrates, while their meeting and falling in love is better, and their meeting, falling in love, and conceiving Socrates is better still. In the circumstantial order of explanation, we look for the circumstances that explain the existence of Socrates, and thereby allow us to introduce the interpreted sentence “Socrates exists” into different conversational contexts. Looking ahead, given certain facts about human biology, we may suppose that the existence of a particular zygote is at least part of an explanation for the existence of Socrates at any context, and thus the putative essence Socrates bears in this identity can be nominalistically reconstructed by the proof theorist interested in introducing object-language talk meant to codify the explanatory practices of the biologist. Conversely, along the consequential order of explanation we hold individual sentences fixed across varying contexts to see what the resulting contexts better explain. The interpreted sentence “Socrates exists”, together with collateral facts about what “Socrates” denotes, better explains “Socrates is located in space and time” than it does “Plato is a philosopher” or “ $2+2=4$ ”, even though both of these latter things might be justifiably inferred from various contexts that include “Socrates exists”. When explanation is conceived as a process of drawing inferences, these two orders of explanation can be used to define introduction and elimination rules for atomic sentences.

3.3 Introduction and Elimination Rules for Atomic Sentences via Circumstantial and Consequential Explanations

There is today a rich field for developing the notion of an explanation, and four or five theories have come to prominence over the last half century. The *deductive-nomological model*, owed largely to Hempel (e.g., 1965), requires that a law be part of the premises, with the explanandum derived deductively from them. The *causal-mechanical model* (e.g., Woodward 2003, Lipton 2004, Strevens 2008) makes use of explanantia containing causal mechanisms. *Unificationist models* require that explanations facilitate our understanding of the world by integrating disparate domains of knowledge into a more unified worldview (Friedman 1974, Kitcher 1989, Schurz and Lambert 1994, Schurz 1999, Bangu 2017). The *defeasible inference model* (Khalifa et al. 2018, Millson et al. 2018, Millson and Straßer 2019) holds that good explanations are those that remain good when their competing explanations fail. Finally, in recent work John Norton (2003, n.d.) has defended a *material model* of explanation as part of a more general material theory of induction in the sciences: “According [to the material theory of induction], what separates the good from the bad inductive inferences are background facts, the *matter* of the inference, as opposed to its *form*” (n.d. Chapter 1, §2).

Explanations are in general non-monotonic, and explanatory inferences can be reconstructed with non-monotonic proof theories (see Litland 2016 and 2018, Millson et al. 2018, Millson and Straßer 2019, and Poggiolini 2016 and 2018). The proof theorist need not commit him- or herself to any particular theory of explanation, however, and for the purposes of showing how to use a notion of explanation to individuate canonical derivations for atomic sentences, it is enough that some such notion is in play. In Stovall (2020), I use a modified version of the explanatory proof theory of Millson and Straßer (2019) to specify the semantic values of atomic sentences under an intended interpretation. That semantics introduces more complexity than is needed here, and the material theory of explanation offers a natural way of spelling out this idea without having to delve into the formal details. Consequently, while I remain officially neutral on how to conceive of explanatory inferences, I frame the view with the material model in mind. More generally, the conceit of treating all of language from either a proof-theoretic or model-theoretic frame is an oversimplification induced for

simplicity and precision. It is likely that an adequate semantics for atomic declarative sentences requires a model theory specifying their representational and agency-involving truth and action conditions, together with both a broad class of proof-theoretically individuated material introduction and elimination rules, and a narrow class of material explanatory rules of inference (with perhaps more besides).¹⁴

Let $\mathcal{L}_{atom}$ name the set of atoms of $\mathcal{L}$, and let Γ_{atom} range over subsets of $\mathcal{L}_{atom}$. The class of material explanatory derivation systems $\mathbf{E}$ for $\mathcal{L}_{atom}$ is such that each $E_i \in \mathbf{E}$ defines, for each $p \in \mathcal{L}_{atom}$, the set of contexts (interpreted sentences) that materially explain p and the set of context/sentence pairs such that p , together with the context of that pair, materially explains the sentence of that pair. Officially, contexts are interpreted sentences and explanations are explanatory *inferences* from contexts to atoms. These contexts may include logically complex sentences, and for the purposes of this essay the notion of material explanation is stipulated as given. Every $ND_i \in \mathbf{ND}$ can be extended with any $E_i \in \mathbf{E}$, defining a class of combined derivation relations $\mathbf{NDE}$ such that each $ND_i E_i \in \mathbf{NDE}$ is drawn from the Cartesian product of $\mathbf{ND}$ and $\mathbf{E}$. For a given combined derivation system $ND_i E_i$ I will refer to the rules of inference determined by ND_i as the logical fragment of that system, as distinguished from

14. The notion of a material inference is an old one, and for centuries logicians regularly distinguished between the sorts of inferences that are underwritten by the meanings of logical vocabulary and those underwritten by the meanings of non-logical vocabulary. A classical statement is found in Pseudo-Scotus' commentary on Aristotle's *Prior Analytics* (c.1340):

Consequences are divided thus: Some are material, others are formal. A formal consequence is one which holds in all terms, given similar mutual arrangement and form of the terms... A material consequence is one which does not hold in all terms given similar mutual arrangement and form so that the only variation is in the terms themselves. (From Bochenski 1961, pp.191-92; cf. the discussion of Pseudo-Scotus' account of formal and material consequence in Kneale and Kneale 1962, pp.278-81)

Compare with Gottfried Ploucquet's 1763 *Methodus Calculandi in Logicis*:

[B]y nature and according to logical order every calculus comes after the understanding of the matter to which the calculus is applied... He who invents does not begin from a calculus, but from the consideration of things. (From Capozzi and Roncaglia 2009, p.135.)

the rules of the explanatory fragment determined by E_i . With the exception of the transitivity principle introduced below, and the notions of immediate and mediate explanation it governs, I place no formal constraints on explanation or the interaction between the explanatory and the logical fragments of combined derivation systems.

For any given $ND_i E_i$, the circumstantial order of explanation in E_i associates each atomic sentence p with a set of contexts that, under that interpretation, warrant introducing p as the conclusion of an explanatory inference. That is, the explanatory fragment E_i for a language $\mathcal{L}_{atom}$ lays down an intended interpretation for the atoms p of $\mathcal{L}_{atom}$ by specifying, for each p , a set of circumstantially explanatory inferences to p . These explanatory inferences are *rules* for inferring to the atoms on the basis of their meanings (comprehensions) under that interpretation.

In this essay, I define introduction rules for atomic sentences on the basis of the explanatory fragments of the combined derivation systems in stages, by first distinguishing immediate and mediate circumstantial explanations. This distinction in explanation will be used to interpret the putatively metaphysical distinction between mediate and immediate essence provided in part 4, but it is worth pointing out that the explanatory introduction and elimination rules provided below can be defined in terms of explanation *simpliciter*. Henceforth, I tend to suppress reference to particular natural deduction and explanatory systems. To avoid staking out positions on explanation that are orthogonal to this discussion, I generally speak of a context as *involving* various sentences in the explanation of other sentences. The official interpretation is that, as a linguistic object of study, an explanation is an inference from a set of premises to a conclusion (questions regarding the nature of inference and explanation turn on whether the premises can be empty, but I consider only nonempty premises here).

There are two ways that a context Γ can be circumstantially explanatory for p . First, the sentences of Γ (or the set of sentences arrived at by the closure of Γ under the logical rules of inference – I suppress this in what follows) explain p . Call this an *immediate circumstantial explanation* for p , or $Ex_I^{Cir}(p)$. Second, a context may be such that it explains other sentences that, together with that context, circumstantially explain p . Call the set of such contexts the

mediate circumstantial explanations for p, or $Ex_M^{Cir}(p)$. If we take the notion of an immediate circumstantial explanation as basic, we can introduce both mediate circumstantial explanations and the general class of circumstantial explanations as derived notions as follows. For each atomic sentence p , we define the set of *circumstantial explanations for p*, or $Ex^{Cir}(p)$, as the set of contexts such that every immediate circumstantial explanation for p is a circumstantial explanation of p , and which is closed under the principle that anything which is circumstantially explained by a context can be used in explaining other things at that context (I number the three conditions of the second element of the union for ease of parsing):

DEFINITION 2 (*Circumstantial Explanations*)

$$Ex^{Cir}(p) =_{def} \{\Gamma : \Gamma \in Ex_I^{Cir}(p)\} \cup \{\Gamma : \text{for some } \Delta_1 \dots \Delta_n$$

1. $\Delta_1 = \Gamma \cup \{q : \Gamma \in Ex_I^{Cir}(q)\}$
2. for each Δ_i such that $1 \leq i \leq n$,
 $\Delta_{i+1} = \Delta_i \cup \{q : \Delta_i \in Ex_I^{Cir}(q)\}$
3. $\Delta_n \in Ex_I^{Cir}(p)\}$

The second element of the union in this definition says that the circumstantial explanations for an atom include any context that can be expanded by deriving its immediate explanations and using them to derive further immediate explanations from the expanded context, with the final context being an immediate circumstantial explanation for the atom. This amounts to imposing a Cut or transitivity principle on explanatory inferences:¹⁵

$$\text{if } \Gamma \in Ex^{Cir}(q) \text{ and } \{\Gamma, q\} \in Ex^{Cir}(p), \text{ then } \Gamma \in Ex^{Cir}(p)$$

We then define the *mediate circumstantial explanations for p*, or $Ex_M^{Cir}(p)$, as the complement of the immediate circumstantial explanations for p in the class of circumstantial explanations for p :

DEFINITION 3 (*Mediate Circumstantial Explanations*)

$$Ex_M^{Cir}(p) =_{def} \{\Gamma : \Gamma \in (Ex^{Cir}(p) - Ex_I^{Cir}(p))\}$$

15. Though not uncontroversial, the explanatory proof systems of Litland (2017) and Millson and Straer (2019) each include a Cut rule.

We now stipulate that the contexts figuring in the introduction rules for atomic sentences under an intended interpretation are those that are circumstantially explanatory for it:

DEFINITION 4 (*Introduction Rules for Atomic Sentences*)

$$\frac{\Gamma}{p} \text{ is an introduction rule for } p \text{ iff}_{def} \Gamma \in Ex^{Cir}(p)$$

Returning to the example from section 3.2, where the intended interpretation is English we can now account for whether “the match is lit” would have:

$$\frac{\text{the match is struck, oxygen is present}}{\text{the match is lit}}$$

as an introduction rule – it does so just in case we accept the fact that the match is struck and there is oxygen present as a circumstantial explanation for the fact that it is lit.

Following the tradition in proof theory, the proposal below focuses on introduction rules. But any time (the truth of) a *sentence* is explained at a context, that context explains (the truth of) the sentence. This fact might be leveraged to take either the circumstantial or the consequential order of explanation as primary, and so reduce either atomic introduction to atomic elimination rules or *vice versa*. In the interest of illustration, I discuss (one version of) the consequential order of explanation and the corresponding individuation of elimination rules. For each atomic sentence p under an intended interpretation, consider the set of context/sentence pairs $\langle \Gamma, q \rangle$ such that Γ (the premises) together with p is a consequential explanation for atomic sentence q (the conclusion). Where $Ex^{Con}(p)$ names this set of consequential explanations for p , we have:

DEFINITION 5 (*Elimination Rules for Atomic Sentences*)

$$\frac{\Gamma, p}{q} \text{ is an elimination rule for } p \text{ iff}_{def} \langle \Gamma, q \rangle \in Ex^{Con}(p)$$

4. Talk of Essence as the Expression of Explanation

4.1 Criteria of Adequacy

There is a growing literature on the use of proof-theoretic explanatory meta-languages to define rules for the use of various object-language vocabularies.

Barker (2012) proposes an expressivist analysis of object-language truth-making claims in terms of a metalanguage of rules of inference. Millson and colleagues (Khalifa et al. 2018, Millson et al. 2018, and Millson and Straßer 2019) introduce a “best explains why” operator – meant for codifying scientific explanation – into an object-language on the basis of a derivation system with non-monotonic explanatory inferences. Developing ideas put forth by Bolzano, Poggiolini (2016 and 2018) provides a sound and complete derivation system employing formal (or logical) explanatory inferences and shows how to introduce an object-language “because” operator on that basis. The aim of this operator is to allow for object-language claims about the explanatory inferences that are employed in the metalanguage. Litland (2018) does something similar by employing the notion of a *metaphysically explanatory argument* to give rules of inference for factive and non-factive metaphysical grounding operators. In each case, proof-theoretic metalinguistic resources are used to define rules for object-language linguistic devices.

If the explanatory inference is used to codify object-language talk of essence, a litmus test for such a proof system is its ability to account for the distinctions that have been drawn in the literature on essence by showing how to proof-theoretically reconstruct them. Here I focus on three sets of distinctions. Two are drawn from Fine (1995c and 1995b) and one from Correia (2006), though all have become part of the common framework for thinking about essence today.

The first of Fine’s distinctions concerns the difference between restricted consequential and unrestricted consequential essence, where the former concerns those things that are, in Fine’s terminology, true in virtue of the nature of an object or collection of objects, while the latter is arrived at by the closure of restricted consequential essence under logical consequence (cf. Fine 1995c, pp.56-61 and 1995b, pp.277-80, and Koslicki 2012; Fine 1995c also uses “constrained” and “unconstrained” to mark this distinction). For instance, being such as to contain Socrates is (we may suppose) part of the restricted consequential essence of {Socrates} insofar as it pertains to what {Socrates} is, whereas being such that $2+2=4$ is part of {Socrates}’s unrestricted consequential essence. The second of Fine’s distinctions concerns that between mediate and immediate essence (cf. 1995c, pp.61-62 and 1995b, pp.280-82). With this distinction, and a transitivity principle across mediation, Fine can hold the following: If Socrates

is part of the immediate essence of {Socrates}, while {Socrates} is part of the immediate essence of {{Socrates}}, then Socrates is part of the mediate essence of {{Socrates}}.

Finally, work that traces to Correia (2006) distinguishes objectual essence as those features of an *object* (or collection of objects) that pertain to what that object is (or what the members of that collection are), from generic essence as those features of a *property* that are true in virtue of what it is to possess that property. Conventional philosophical wisdom today has it that we can sensibly talk about whether water is essentially H_2O , or whether the substance water, or some quantity of it, could have been something else. I use “characteristics” interchangeably with “properties”, and I mean to be neutral with regard to the underlying ontology. Relations of ontological dependence among objects and characteristics include cases where objects are essential to objects (Socrates and {Socrates}); cases where characteristics are essential to characteristics (being water and being a molecule); cases where characteristics are essential to objects (Socrates and his humanity); and cases where objects are essential to characteristics (Socrates and his haecceity, or the property he has in virtue of being the very individual he is). Given the contentious status of the fourth class, and its comparatively rare mention in the contemporary literature, its discussion is omitted here. Additional subdivisions may be necessary, and it is an open question whether the proof theorist will be able to account for every permutation. Furthermore, as will become clear, features specific to different classes of object and characteristic require specifying the account in various ways.

I return to these issues in section 5.2. In the interim, the discussion is framed around the distinction between objectual and generic essences in section 4.2, section 4.3, and section 4.4. Cases of immediate and mediate essence are handled along the way, and a discussion of unrestricted and restricted consequentialist essence occurs in section 4.5. In section 4.6, I briefly examine colloquial talk of essence. It is worth emphasizing that the proof theorist need not think any of these distinctions carve nature at its joints (to stick with an ontological turn of phrase) in order to profitably investigate their use in the inference games we play with one another.

4.2 Objectual Essence: Ontological Dependence Among Objects

With the resources introduced above, and with reference to a particular $ND_i E_i \in \mathbf{NDE}$ that provides the interpretation in question, an assertion of the object-language sentence:

{Socrates} is essentially such that Socrates exists

can be understood as expressing commitment to the metalinguistic claim that for every Γ that licenses a circumstantially explanatory inference to “{Socrates} exists”, either “Socrates exists” is part of Γ , or “Socrates exists” licenses a circumstantially explanatory inference to some sentence in Γ :

for every $\Gamma \in Ex^{Cir}(\text{“{Socrates} exists”})$
 either “Socrates exists” $\in \Gamma$ or
 for some $p \in \Gamma$, “Socrates exists” $\in Ex^{Cir}(p)$

The first disjunct covers those cases where Γ is an immediate circumstantial explanation for “{Socrates} exists”, and the second disjunct covers cases where Γ is a mediate circumstantial explanation. The existential quantification is to allow for partial mediate explanations, as when Socrates is said to be essential to {Socrates, Plato} – in this case, the fact that Socrates exists is a mediate explanation for the fact that {Socrates} exists, which itself is only part of the explanation for the existence of {Socrates, Plato}.

Notice that the metalinguistic claim concerns sentences rather than the facts they denote in the object language. If we allow ourselves the device of semantic descent in the metalanguage (as in the explanation for the existential quantification just given), then this analysis can be reformulated with talk of facts rather than interpreted (sets of) sentences. In this case, an assertion of “{Socrates} is essentially such that Socrates exists” is interpreted as an expression of the assertion that, for every set of facts Γ that is a circumstantial explanation for the fact that {Socrates} exists, either the fact that Socrates exists is part of Γ , or the fact that Socrates exists is a circumstantial explanation for some fact in Γ . I speak of explanatory connections among interpreted sentences in the interest of minimizing the use of ontological language, but it is important to note that even with an ontology of facts there is no need to use a metalanguage of essences to account for what object-language talk of essence amounts to on this analysis:

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All one needs are facts and their explanations.

More generally, to say that some individuals $s_1, s_2, \dots$ are each individually essentially such that some other individuals $t_1, t_2, \dots$ exist is to express commitment to the claim that any circumstantially explanatory inference to “ s_i exists”, where “ s_i ” $\in \{“s_1”, “s_2”, \dots\}$, will have as its premise context a set of sentences such that, for every “ t_i exists” $\in \{“t_1$ exists”, “ t_2 exists”, $\dots\}$, either “ t_i exists” $\in \Gamma$ or “ t_i exists” licenses a circumstantially explanatory inference to some p in Γ . That is, any circumstantial explanation for the fact that any s_i exists will, for each t_i , either include the fact that t_i exists, or will be such that the existence of t_i explains some fact in that explanation. Relative to an interpretation:

DEFINITION 6 (Essential Relations Among Objects)

the object-language claim

$s_1, s_2, \dots$ are (individually) essentially such that $t_1, t_2, \dots$ exist

gives expression to the metalinguistic claim

for every “ s_i ” $\in \{“s_1”, “s_2”, \dots\}$ and every “ t_i ” $\in \{“t_1”, “t_2”, \dots\}$ and every $\Gamma \in Ex^{Cir}(\text{“}s_i \text{ exists”})$
 [either “ t_i exists” $\in \Gamma$ or, for some $p \in \Gamma$, “ t_i exists” $\in Ex^{Cir}(p)$]

On this proposal, then, to say that some object is essential to another object is to express commitment to the context-invariant goodness of the explanation for the existence of the second on the basis of the existence of the first, and so to give expression to structural features of the introduction rules for atomic claims about different objects. Call this sort of explanation a *universal circumstantial explanation*. Where explanatory inferences provide the semantic values of atomic sentences, talk of essential ontological dependence thus interpreted is a device for talking about the language under a given interpretation without having to adopt an explicitly metalinguistic vocabulary.

A natural worry is that this analysis merely pushes the question that the metaphysician is interested in – viz., what it means to attribute essences to individuals – back to the level of the explanations employed in the metalanguage as a basis for individuating the introduction and elimination rules of object-language sentences. This is particularly pressing given the symmetric

metaphysically necessary relation between the existence of Socrates and the existence of {Socrates}. Thus, the question

Why is the existence of Socrates essential to the existence of {Socrates} rather than *vice versa*?

becomes

Why is there an introduction rule for “{Socrates} exists” that appeals to “Socrates exists” rather than *vice versa*?

which on the current analysis is equivalent to the question

Why is the circumstantial explanation for the existence of {Socrates} on the basis of the existence of Socrates acceptable, but not *vice versa*?

This may give the appearance of smuggling metaphysical realism back into the picture. But an answer to this question can be given without appeal to anything but the practice of identifying and individuating sets and their members, which paves the way for the proof-theoretic nominalist to reconstruct a logic of essence on the basis of a reflective examination over first-order inquiry.

To see this, consider the reasons one gives for claims like “ s_1 is the same set as s_2 ” and “ t_1 is the same human being as t_2 ”. In giving a reason for making a claim of the former sort, we refer to the individuals that constitute the members of the sets s_1 and s_2 . But in giving a reason for saying the latter we make no reference to the sets $\{t_1\}$ and $\{t_2\}$. For instance, the set of prime numbers between 3 and 7 (inclusive) is the same set as the set of odd numbers between 3 and 7 (inclusive), and this can be shown by enumerating the members of the two sets under their respective characterizations and establishing that all and only the numbers 3, 5, and 7 occur in both enumerations. In doing that, we employ no talk of what it is to be a number or set – we simply verify that both sets contain the same numbers. And doing *that* turns on whatever capacity we have to identify and individuate numbers and sets – if this presupposes some grasp of essence, it is not evident in anything we do or say. Consequently, if the objection is raised that there is some hidden reliance on essence at work in listing the odd numbers from 3 to 7 inclusive, listing the prime numbers from 3 to 7 inclusive,

and showing that the same terms occur on each list, the objector owes us some account of where that reliance is evident. In justifying the claim that Socrates is the same human being as the most famous teacher of Plato, however, we make no reference to the sets containing these individuals. Instead, we trace the life of the human being Socrates so as to show that he is the very same human being that is the most famous teacher of Plato. Our practice of justifying the assertion of sentences that identify sets proceeds by way of the identification of their members, but we do not justify the assertion of sentences that identify human beings by an appeal to the sets containing them.¹⁶

Because circumstantial explanations were used to define introduction rules for atomic sentences, object-language talk of the essences of individuals gives expression to a structural feature of the rules that govern sentences concerning the existence of individuals, which rules are in turn individuated via whatever explanations we go in for. Object-language metaphysical talk is interpreted in a metalinguistic explanatory register – the *essentiality* of Socrates for {Socrates} is understood in terms of the *explanatory connection* between the individuation of {Socrates} and the individuation of Socrates, as exhibited in the uses to which we put the terms “{Socrates}” and “Socrates”.

With the distinction between immediate and mediate circumstantial explanations, it is possible to account for the difference between immediate and mediate objectual essences as well. Consider the claim that Socrates is mediately essential to {{Socrates}}. Insofar as {{Socrates}}’s existence is immediately explained by {Socrates}’s existence rather than Socrates’ existence, the essential relation between Socrates and {{Socrates}} will be mediated by the essential relation between Socrates and {Socrates}, and the essential relation between {Socrates} and {{Socrates}}. That is to say, insofar as the sentence “Socrates exists” is part of any immediate circumstantial explanation for the sentence “{Socrates} exists”, while the latter is part of any immediate circumstantial explanation for the sentence “{{Socrates}} exists”, then via the transitivity principle of circumstantial explanations it follows that “Socrates exists” is a

16. We could of course adopt a convention whereby we identify individuals according to the sets they occur in; but to do *that* would be to change the language, and where the semanticist is interested in characterizing a language as it is used, such a possibility is no objection to the characterization on offer.

mediate circumstantial explanation for “ $\{\{Socrates\}\}$ exists”. Whether or not our explanations are fine-grained enough to admit a clear distinction between immediate and mediate explanation is a different issue, of course.

4.3 Generic Essence: Ontological Dependence Among Characteristics

Now consider generic essences of the form:

F s are essentially G

or

to be F is essentially to be G

A number of different kinds of cases can be individuated here, but I will focus on two. On one hand, there are cases where talk of an essential characteristic either gives voice to an identity between the possession of one characteristic and the possession of the other (“to be water is essentially to be H_2O ”) or which concern part/whole relations (“water is essentially such as to contain hydrogen”); I focus on chemical wholes, although, so far as I can see, similar remarks hold for other kinds of containment relations). On the other hand, claims like “humans are essentially mammals” and “water is essentially a molecule” concern species/genus relations (in the Aristotelian rather than biological sense). The two cases require different treatment, and other sorts of characteristic-characteristic relations (e.g., between kinds and their properties, or between determinations and determinables) may call for additional analyses.

4.3.1 Essential Characteristic Identities and Part/Whole Relations

To say that to be F is essentially to be G , where “ F ” and “ G ” are either predicates or sortals (mass or count nouns) and where the copula indicates either an identity or a part/whole relation, is to express commitment to the claim, general for “ t ”, that any circumstantially explanatory inference for “ t is F ” will involve “ t is G ”.

DEFINITION 7 (*Essential Relations Among Characteristics*)

where F either contains or is identical to G , the object-language claim

to be F is essentially to be (or contain) G

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gives expression to the metalinguistic claim

for every “ t ” and for every $\Gamma \in Ex^{Cir}(\text{“}t \text{ is } F\text{”})$
 $[\text{either “}t \text{ is } G\text{”} \in \Gamma \text{ or } \Gamma \in Ex^{Cir}(\text{“}t \text{ is } G\text{”})]$

To say that water is essentially H_2O , then, is to give expression to the claim that any context that explains why something is water will involve either appeal to the fact that it is H_2O , or will itself be a context that explains why it is H_2O . Similarly, to say that water is essentially such as to contain hydrogen expresses commitment to the metalinguistic claim that anything that explains why something is water will either involve the claim that it contains hydrogen, or will explain why it contains hydrogen. And so talk of the essence of water as H_2O , or as containing hydrogen, can be understood to give expression to an explanation for why something is water at any context.

As to why these explanations hold, consider how we identify and individuate quantities of water. In practice, one can be taught to reliably (though fallibly) sort liquids according to whether or not they are water. On this basis, one can (defeasibly) infer the sentence “this stuff is water” from an observation sentence like “this stuff is a clear, odorless, tasteless liquid” at many contexts. The world is not such as to ensure that our pre-scientific habits of classification are universally reliable, however. To be told “water is essentially H_2O ” allows one to expand one’s stock of accepted inferences with the corresponding universal (context-invariant) explanatory inference. Now, whatever defeasible criteria of classification one used for identifying water can be replaced with a (putatively) indefeasible one (though much harder to apply in most situations we find ourselves in). And so if one accepts that the inference from “this stuff is H_2O ” to “this stuff is water” is explanatory in every context, then talk of the essence of water as H_2O can be understood as a means of substituting defeasible criteria of classification for an indefeasible one, where the only grasp of the world we must have is whatever grasp we already have on stuffs and their molecular compositions. Once again, rather than presupposing a grasp of essence, the ordinary practice of identifying and individuating objects in the world is sufficient for introducing essentialist claims into the object-language as a means of better equipping practitioners of the language to reason across different background conditions.

Concerning the relationship between immediate and mediate generic essences, this again turns on what sorts of explanations a proof theorist admits as immediate. For instance, suppose that the liquid that falls from the sky and is in the earth's oceans, lakes, and rivers is essentially water. It follows that the liquid that falls from the sky, etc. is essentially H₂O as well. But is this because being H₂O mediately explains what the liquid that falls from the sky, etc. is by immediately explaining what water is; or is that former explanation immediate? As with the immediate/mediate distinction in the case of sets and their members, I do not see that our habits of explanation concerning water, H₂O, and the liquid that falls from the sky are sufficiently precise as to settle this question. This situation is complicated by the fact that H₂O is a classification that has been introduced on the basis of scientific discovery, which in turn has revised the way we understand the world. But insofar as the philosopher is interested in immediate and mediate essences, this is how to proof-theoretically and nominalistically introduce them into an object language governed by the corresponding explanatory inferences.

4.3.2 *Essential Relations Among Species and Genera*

Some essential relations among general characteristics relate being *F* and being *G* as species to genus: e.g., "H₂O is essentially a molecule" (I shift to talk of the molecule rather than the molecular stuff here). Going off the pattern of the cases thus far, one would expect this claim would give expression to the universal circumstantial explanation for "*t* is H₂O", general for "*t*", involving use of "*t* is a molecule". But being a molecule does not explain something being H₂O, as many things are molecules without being H₂O. But in the case of species/genus relations, we reason according to the principle that the species inherits all of the properties that are essential to the genus, and this is reflected in a peculiar feature of the explanations that we can give of species in terms of their genus. For instance, all molecules are bound together by covalent bonds which, containing no free electrons, make them electrically neutral. The fact that H₂O is electrically neutral can therefore be explained, we may suppose, by the fact that H₂O is a molecule (pure water does not conduct electricity; it is impurities in water that, as ions having free electrons, enable it to conduct electricity).

This points the way toward the following interpretation: The essentialist species/genus claim "to be *F* (species) is to essentially be *G* (genus)" expresses commitment to the claim that, where there is a circumstantially explanatory inference from "*t* is *G*" to "*t* is *H*" (general for "*t*" and for "*H*"), there is a circumstantially explanatory inference from "*t* is *F*" and "to be *F* is to be *G*" to "*t* is *H*".

DEFINITION 8 (*Essential Relations Among Species and Genera*)

where *F* is a species of *G*, the object-language claim

to be *F* is essentially to be *G*

gives expression to the metalinguistic claim

for every "*t*" and every "*H*" s.t. "*t* is *G*" is in some $\Gamma \in Ex^{Cir}("t \text{ is } H")$
 $[(\{"t \text{ is } F"\} \cup \{"to \text{ be } F \text{ is to be } G"\} \cup \Gamma - \{"t \text{ is } G"\}) \in Ex^{Cir}("t \text{ is } H")]$

Thus, the claim that H₂O is essentially a molecule functions as an inference ticket licensing one to make an explanatory inference, for any characteristic *H* that is explained by being a molecule, to the claim that *t* is *H* (general for *t* and *H*) from any context that includes the claims that *t* is H₂O and that H₂O is a molecule. The need to include reference to the species being of the genus is to ensure that the explanatory connection between being of the genus and bearing the characteristic is preserved. For where being a molecule explains being electrically neutral, it is not being composed of H₂O by itself that explains some body of H₂O being electrically neutral; it is rather being composed of H₂O together with H₂O being a molecule that underwrites this explanation. And the requirement that being *G* figure only in *some* circumstantial explanation for being *H* is to allow that there are non-*G* explanations for being *H* (e.g., where *H* is a multiply realizable property) – what is important is that, where being a genus *G* is essential to being a species *F*, anything explained by being *G* will also be explained by being *F*, together with the claim that *F*s are *G*.

There is a question of whether this should be seen as a kind of immediate/mediate essential relation distinct from those considered thus far. Where being a molecule is essential to being H₂O, does it follow that being electrically neutral is part of the essence of H₂O because H₂O is essentially a molecule?

On one hand, when gripped by the convictions of an essentialist, reflection on the question suggests the answer is “yes”. On the other hand, if the claim that H_2O is essentially a molecule is a claim that identifies H_2O with a (kind of) molecule, it is not clear that the (putative) essential relationship between being H_2O and being electrically neutral, “mediated” by the fact that H_2O is essentially a molecule, is of the same sort as the relationship between Socrates and $\{\{\text{Socrates}\}\}$, mediated by the fact that Socrates is essential to $\{\text{Socrates}\}$, and $\{\text{Socrates}\}$ is essential to $\{\{\text{Socrates}\}\}$ – for the relationship between an individual and the singleton set of that individual is not an identity between two kinds. It may be that further reflection on the way we reason about individuals and kinds will help settle this issue.

4.4 *Ontological Dependence of Individuals on Characteristics*

On an adverbial analysis, the claim “humanity is essential to Socrates” goes over into “Socrates is essentially such that he is human” or “Socrates is essentially human”. The question now is what being human, possessed by an individual, explains. Being human does not explain the existence of Socrates, insofar as his existence is explained by features of his parents, etc. But being human does explain various properties any human possesses. For instance, the fact that Socrates is a human being, together with the claim that a human being is essentially such as to be identical with a zygote, explains why Socrates is identical with a zygote. As with the case of genera/species relations of essence, this suggests that the claim that Socrates is essentially human expresses commitment to the claim that the fact that someone is Socrates explains everything about that individual that is explained by his being human (this is not to deny that being Socrates explains *more* than what is explained by being human, of course):

for every “ t ” and “ F ” and Γ s.t.
 “ t is human” $\in \Gamma$ and $\Gamma \in Ex^{Cir}(\text{“}t \text{ is } F\text{”})$
 $(\{\text{“}t = \text{Socrates”}\} \cup \{\text{“to be human is to be } F\text{”}\} \cup \Gamma - \{\text{“}t \text{ is human”}\}) \in Ex^{Cir}(\text{“}t \text{ is } F\text{”})$

More generally:

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DEFINITION 9 (*Characteristics Essential to Objects*)

the object-language claim

s is essentially H

gives expression to the metalinguistic claim

for every “ t ”, “ F ”, Γ s.t. “ t is H ” $\in \Gamma$ and $\Gamma \in Ex^{Cir}(\text{“}t \text{ is } F\text{”})$
 $[(\{\text{“}t = s\text{”}\} \cup \{\text{“to be } H \text{ is to be } F\text{”}\} \cup \Gamma - \{\text{“}t \text{ is } H\text{”}\}) \in Ex^{Cir}(\text{“}t \text{ is } F\text{”})]$

Once again, the inclusion of a claim linking being H to being F is to preserve the explanatory connection that talk of essence goes proxy for. Here a plausible case can be made, I think, that we speak as if the essential characteristics that Socrates inherits in virtue of being human are part of the immediate essence of humanity and the mediate essence of Socrates. For being Socrates explains why someone is a rational animal, for instance, only because Socrates is human and humans are rational animals. Once again, however, facts about the explanations we go in for (and of how we formalize common nouns) may push us in one direction or another.

4.5 *Unrestricted and Restricted Consequential Essences*

At this point, I have argued that a proof-theoretic nominalist account of essence can capture the distinction between immediate and mediate essences, and a variety of objectual and generic essences. The third desideratum was to account for the distinction between unrestricted and restricted consequential essences. In the interest of space, I will focus this discussion on objectual essences; related remarks go over for the other cases discussed above.

Intuitively, not every part of an object’s consequential essence will pertain to the nature of that object, for otherwise every logical truth will be part of the essence of every object. The most common method of recovering a notion of consequential essence whose features are relevant to the object in question is to begin with an unrestricted notion of consequential essence that includes irrelevant consequences, and define a notion of restricted consequential essence by generalizing out the logical consequences that follow from any object’s essence whatsoever (see, e.g., Fine 1995c, p.59, Fine 1995b, pp.277-8, and Koslicki 2012, pp.192-93).

On the current proposal, the unrestricted logical consequences of an object's essence do not count as part of that object's essence to begin with. For on this proposal, an essential property is one that underwrites universal circumstantial explanations, and explanations are sufficiently fine-grained as to rule out these sorts of cases. A circumstance containing only the sentence "Socrates exists" by itself entitles an explanatory inference (we may suppose) to "{Socrates} exists". As a consequence, the sentence " $2+2 = 4$ " is not part of every context that explains why {Socrates} exists. From this it follows that the sentence "{Socrates} is essentially such that $2+2 = 4$ " does not give expression to a universal circumstantial explanation for the existence of {Socrates} – where the intended interpretation for the language is the English we currently speak. Nevertheless, at first glance it may look like there can be cases where an object bears a characteristic in every circumstance in which it exists that is explanatory in every such circumstance. For Socrates is such that there are four primes between one and ten, and this is a property he bears in every context in which he exists. Furthermore, this property looks to explain why *Socrates is such that there are more than three primes between one and ten*. But surely we do not want to say that it is essential to Socrates that there are four primes between one and ten.

To appreciate that this is not a counterexample to the proposal, consider how to go about explaining why Socrates is such that there are more than three primes between one and ten. What explains this fact is the pair of facts that 1) Socrates exists, and 2) there are four primes between one and ten. And *anything* is such that there are four primes between one and ten; what is doing the explanatory work is not the existence of an object with the complex characteristic "being such that. . .", but rather certain facts about the natural numbers together with the fact that the object exists. One might next wonder whether we would thereby be committed to the claim that it is essential to the joint fact *that Socrates exists and that there are four primes between one and ten* that Socrates is such that there are more than three primes between one and ten. That commitment is optional, depending on whether we accept the truth of "Socrates exists and there are four primes between one and ten" as a universal circumstantial explanation for the truth of "Socrates is such that there are more than three primes between one and ten". It is not clear to me that this rarefied philosophical example is such

as to be settled by the explanations we habitually employ, and either way the present proposal is not committed to a view on which the logical consequences of an object's essence are part of that object's essence.

4.6 Non-Philosophical Uses of Essence

The account of explanation introduced in part 3 was directed at explaining both the universal circumstantial explanations characteristic of philosophical uses of essentialist vocabulary, and the kinds of explanations that underwrite non-philosophical uses of that vocabulary, as in:

When the only good quarterback on a team without a running game gets hurt, the game is essentially over.

and

In this company, a good work ethic is essential to success.

Because the explanations introduced as the basis of the semantic values of atomic sentences in section 3.3 are context-relative, it follows that non-philosophical use of essentialist language can be understood as expressing commitment to general but non-universal explanatory relations between, e.g., passing, running, and winning in football, and work ethic and professional success. Philosophical talk of essence thus appears as a special case of the general role of essentialist vocabulary as a modal vocabulary concerning, to put the point in Kantian terms, the value of the copula in relation to thinking in general.

5. On the Prospects of Proof-Theoretic Modal Metaphysics

5.1 Summary

At the start of the essay, I claimed that some of the commitments animating the project of model-theoretic realist analytic metaphysics appear optional from the standpoint of developments in proof theory. Whereas realism about metaphysical vocabulary has been the predominant stance in Anglophone analytic metaphysics since roughly the middle of the 20th century, manifest in often-unquestioned use of metaphysical language to specify truth conditions in model-theoretic metalanguages, I said that proof theory could be used to illuminate the terrain of metaphysics – and the cognition involved in metaphysical speculation – from

nominalist points of view. The intervening discussion has tried to make good on these claims by presenting a proof-theoretic nominalism about essentialist vocabulary.

I began by showing that a proof-theoretic metalanguage of explanation affords an interpretation of the semantic values of atomic sentences on the basis of their roles in explanation. This in turn allows for a nominalist interpretation of a range of object-language mediate and immediate, restricted consequentialist and unrestricted consequentialist, essentialist vocabularies concerning relationships between individuals, between characteristics, between individuals and characteristics, between parts and wholes, and between genera and species. On this analysis, claims about essence are understood as devices for expressing commitment to structural features of a proof-theoretic metalanguage: Overtly metaphysical object-language talk of essence can be understood as covertly metalinguistic talk of context-invariant explanations and the rules of inference they determine. Understood in this way, object-language talk of essence enables critical examination of the rules of explanatory reasoning we habitually employ about some domain of inquiry, but without having to adopt the explicitly metalinguistic resources of rule and inference. As to why those rules are as they are, we look to first-order inquiry into the domain at issue in order to determine which explanations are endorsed. *Metaphysical* debate about essence can then be reconceived as *metalinguistic* debate concerning how to set up the rules that govern a language under an intended interpretation, which has its ground in the explanations we seek and give of the various things we are interested in understanding. And so the question of whether essences *exist* is replaced with a question concerning whether our *practices* are such that we are committed to anything like the philosophical notion of essence.

This alternative understanding of the study of (our thought about) essential relations gives metaphysicians tools for considering alternatives to some of the realist and representational presuppositions operating in the background of ongoing debates. Analytic metaphysics developed on the basis of the refinement of a particular set of resources worked up by mathematicians, logicians, and philosophers over the course of the 20th century. As the extensional and realist lens of metaphysical discourse was given priority in philosophers' views, that side of linguistic meaning came to dominate the way we think about things

like meaning and thought. At the same time, the construction of metaphysical theories appealing to ever-more-recherché ontological postulates appears to be an artifact of model-theoretic notions of meaning. Only with a better understanding of the resources of proof theory will it be possible to explain why nominalism, and a more dutiful examination of the explanatory practices of the various sciences, should not temper the excesses of model-theoretic realist speculation.¹⁷

In closing out the essay, I discuss some open questions and offer some brief remarks on extending this program beyond the proof-of-concept developed here.

5.2 Open Questions

I have had to make simplifying assumptions at various points in order to present the core details in the span of an essay of this length. In some cases, the complexities lying behind those assumptions remains to be examined. I will mention two classes of question left open by this analysis: The first concerns details of the underlying proof system, and the second concerns the use of that proof system to interpret talk of essence.

The notion of explanation advanced in part 3 appealed to a material theory of explanation, but as noted there, a number of theories of explanation are on offer in the literature today. And in section 3.3, circumstantial and consequential explanations were defined only for the atoms of the language. Consideration of the explanatory comprehensions for logically complex sentences requires an excursus on rules for drawing logically complex explanatory consequences from bodies of information. Poggiolesi's work on formal explanatory proof systems

17. This should not be read as implying that all of analytic metaphysics is defined by this characterization, and the views of philosophers like Barker, Brandom, Dummett, Litland, Millson, and Poggiolesi illustrate that proof theory has not been entirely absent from the broader philosophical culture within which analytic metaphysics has developed. In addition, expressivist research in metaphysics – as in the work of Flocke, Hofweber, Sellars, and Thomasson – indicates that the realism motivating so much of analytic metaphysics has not gone entirely unquestioned. But in terms of trends or tendencies, there can be no doubt that model-theoretic realism has been the primary lens for metaphysical study in this tradition since roughly Carnap's turn to state descriptions and truth-conditional semantics in the late 1930s (Peregrin 2020 and Tuboly 2017 show that Carnap's so-called "syntactic" period was already semantical, if semantics is understood inferentially; in this regard it is better to see Carnap as moving not from syntax to semantics, but from proof-theoretic semantics to model-theoretic semantics).

might be used as a basis for that excursus. It is also clear that a non-explanatory notion of material inference is needed to account for the full comprehensions of sentences. For the inference from “Prague is east of Paris” follows from “Paris is west of Prague” on the basis of what “east” and “west” mean (in some sense), but there is no *explanation* for one fact on the basis of the other. Here it may be worthwhile to draw on work in the history of logic, both concerning the extension/comprehension distinction (mentioned in section 1.1) as it has been variously conceived, and concerning the distinction between formal and material inference (mentioned in note 14).

Concerning the account of essence developed in part 4, it is an open question whether the proof theorist can account for all the distinctions of essence that have been advanced in the literature. There are relationships between kinds and their properties, genera and their species, and determinables and their determinations that I have left unresolved, along with the possibility, raised at the end of section 4.3.2, that there are at least two different kinds of mediate/immediate essence relations. Finally, there is a question of the relationship between essence and metaphysical grounding, given that the latter is widely understood in terms of explanation (cf. the citations in section 1.1).

5.3 *Metaphysical Research as a Reflection on Our Modes of Reasoning*

It is a sign of the productivity of the proof-theoretic approach to metaphysics that these questions remain open. And the analysis given here for essence is not a one-off, as other forms of metaphysical vocabulary admit of proof-theoretic treatment along similar lines. The rules of inference that govern alethic modal vocabulary (concerning ways the world can and cannot be) differ from the rules that govern normative vocabulary (concerning the way an organism or artifact ought and may be), which rules again differ from those governing the deontic modal vocabulary we use to talk about rational thought and agency. These differences can be systematically codified. The strong and weak alethic modal operators, for instance, can be defined in terms of universal and existential quantification over the antecedents of subjunctive conditionals (see chapter 3 of Stovall 2015 and chapter 5 of Williamson 2007):

$$\begin{aligned}\Box\phi &\text{ iff}_{def} \forall\psi(\psi > \phi) \\ \Diamond\phi &\text{ iff}_{def} \exists\psi(\psi > \phi)\end{aligned}$$

In this regard, the alethic modalities can be understood as devices for giving expression to large-scale features of the space of subjunctive reasoning in terms of which we understand the way things behave (this is another instance of the Kantian/Sellarsian/Brandomian notion of modality discussed in section 2.1). And with a specification of the rules governing the use of two-place teleological operators like “so that” and “in order to” – and an understanding of the different ways we reason about organisms, artifacts, and persons as purposive – one can study the categorial structure of our thought about very general kinds like *chemical*, *organism*, *artifact*, and *person* by studying the rules that govern how we infer to and from modal claims made about chemicals, organisms, artifacts, and persons. Study of what it means to *be* a member of one of these kinds is thereby transposed into a self-reflective key, as a study of the way we *reason* about different kinds by cognizing their members under the forms of reasoning associated with different modalities.

Proof-theoretic nominalism does not do away with metaphysics as the study of reality, then, so much as reconceive it as an effort to understand the rules that govern the way we *think* about reality. As a result, metaphysical speculation is reframed as an engineering task in the logic of the sciences.¹⁸

References

- Almog, Joseph. 2003. “The Structure-in-Things: Existence, Essence, and Logic.” *Proceedings of the Aristotelian Society* 103 (1): 197–225.
- Awodey, Steven, and André W. Carus. 2007. “Carnap’s Dream: Gödel, Wittgenstein, and *Logical Syntax*.” *Synthese* 159: 23–45.

18. I develop the ideas discussed in this section more extensively in Stovall (2015). I should also mention that a variety of model-theoretic realist metaphysical speculation is compatible with this analysis, but only because that speculation is directed at a material domain of facts (the nature of human rationality) the sciences concurrently help illuminate. Stovall (Forthcoming A) offers the full exposition of this scientifically tempered model-theoretic metaphysics, though an overview of a central line of argument is presented in Stovall (Forthcoming B), and the formal details are presented in Stovall (Forthcoming C). In future work, I hope to compare that model-theoretic realist metaphysics with the proof-theoretic nominalist metaphysics developed here, by conceiving the semantics of the former in terms of word-world and world-word relations of extension, and conceiving the semantics of the latter in terms of word-word relations of comprehension.

- Ballarín, Roberta. 2017. "Modern Origins of Modal Logic." In *The Stanford Encyclopedia of Philosophy*, Summer 2017, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University.
- Bangu, Sorin. 2017. "Scientific Explanation and Understanding: Unificationism Reconsidered." *European Journal for Philosophy of Science* 7 (1): 103–26.
- Bar-Hillel, Yehoshua. 1963. "Remarks on Carnap's *Syntax of Language*." In *The Philosophy of Rudolph Carnap*, edited by Arthur Schilpp, 519–543. La Salle: Open Court Publishing.
- Barker, Stephen. 2012. "Expressivism about Making and Truth-Making." In *Metaphysical Grounding: Understanding the Structure of Reality*, edited by Fabrice Correia and Benjamin Schnieder, 272–293. New York: Cambridge University Press.
- Belnap, Nuel. 1990. "Declaratives Are Not Enough." *Philosophical Studies* 59 (1): 1–30.
- Bird, Alexander. 2007. *Nature's Metaphysics: Laws and Properties*. New York: Oxford University Press.
- Bochenski, Joseph M. 1961. *A History of Formal Logic*. Edited and translated by Ivo Thomas. Notre Dame: University of Notre Dame Press.
- Brandom, Robert B. 1994. *Making It Explicit: Reasoning, Representing, and Discursive Commitment*. Cambridge: Harvard University Press.
- . 2008. *Between Saying and Doing: Towards an Analytic Pragmatism*. New York: Oxford University Press.
- . 2015. *From Empiricism to Expressivism: Brandom Reads Sellars*. Cambridge: Harvard University Press.
- Braun, David. 2012. "Extension, Intension, Character, and Beyond." In *Routledge Companion to Philosophy of Language*, edited by Gillian Russell and Delia Graff, 9–17. New York: Routledge.

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- Capozzi, Mirella, and Gino Roncaglia. 2009. "Logic and Philosophy of Logic from Humanism to Kant." In *The Development of Modern Logic*, edited by Leila Haaparanta, 78–158. New York: Oxford University Press.
- Carnap, Rudolph. 1947. *Meaning and Necessity: A Study in Semantics and Modal Logic*. Chicago: University of Chicago Press.
- . 2002. *The Logical Syntax of Language*. Translated by Amethe Smeaton. Chicago: Open Court Publishing.
- Carnino, Pablo. 2014. "On the Reduction of Grounding to Essence." *Studia Philosophica Estonica* 7 (2): 56–71.
- Christensen, Ryan. 2014. "Essence, Essence, and Essence." *Studia Philosophica Estonica* 7 (2): 72–87.
- Correia, Fabrice. 2006. "Generic Essence, Objectual Essence, and Modality." *Noûs* 40 (4): 753–767.
- . 2007. "(Finean) Essence and (Priorean) Modality." *Dialectica* 61 (1): 63–84.
- . 2012. "On the Reduction of Necessity to Essence." *Philosophy and Phenomenological Research* 84 (3): 639–653.
- . 2013. "Metaphysical Grounds and Essence." In *Varieties of Dependence. Ontological Dependence, Grounding, Supervenience, Response-Dependence*, edited by Miguel Hoeltje, Benjamin Schnieder, and Alex Steinberg, 271–296. München: Philosophia Verlag.
- Correia, Fabrice, and Alexander Skiles. 2017. "Grounding, Essence, and Identity." *Philosophy and Phenomenological Research* 98 (3): 642–670.
- Cowling, Sam. 2013. "The Modal View of Essence." *Canadian Journal of Philosophy* 43 (2): 248–266.
- Dasgupta, Shamik. 2014. "The Possibility of Physicalism." *The Journal of Philosophy* 111 (9/10): 557–592.

- Dummett, Michael. 1981. *Frege: Philosophy of Language*. 2nd Ed. Cambridge: Harvard University Press.
- . 1993. *The Logical Basis of Metaphysics*. Cambridge: Harvard University Press.
- Ellis, Brian. 2001. *Scientific Essentialism*. New York: Cambridge University Press.
- . 2002. *The Philosophy of Nature: A Guide to the New Essentialism*. Ithaca: McGill-Queen's University Press.
- Fine, Kit. 1994. "Essence and Modality." *Philosophical Perspectives* 8: 1–16.
- . 1995a. "Ontological Dependence." *Proceedings of the Aristotelian Society* 95 (1): 269–290.
- . 1995b. "Senses of Essence." In *Modality, Morality, and Belief: Essays in Honor of Ruth Barcan Marcus*, 53–74. New York: Cambridge University Press.
- . 1995c. "The Logic of Essence." *Journal of Philosophical Logic* 24 (3): 241–273.
- . 2000. "Semantics for the Logic of Essence." *Journal of Philosophical Logic* 29 (6): 543–584.
- . 2012. "Guide to Ground." In *Metaphysical Grounding: Understanding the Structure of Reality*, edited by Fabrice Correia and Benjamin Schnieder, 37–80. New York: Cambridge University Press.
- . 2015. "Unified Foundations for Essence and Ground." *Journal of the American Philosophical Association* 1 (2): 296–311.
- von Fintel, Kai. 2006. "Modality and Language." In *Encyclopedia of Philosophy*, 2nd Edition, Vol. 10, edited by Donald M. Borchert, 20–27. Detroit: MacMillan Reference USA.
- Flocke, Vera. Forthcoming. "The Language of Ontology." In *The Language of Ontology*, edited by James Miller. New York: Oxford University Press.

- Francez, Nissim. 2014. "The Granularity of Meaning in Proof-Theoretic Semantics." In *Logical Aspects of Computational Linguistics*, edited by Nicholas Asher and Sergei Soloviev, 96–106. Berlin: Springer.
- . 2015. *Proof-Theoretic Semantics: Studies in Logic*, Vol. 57. London: College Publications.
- . 2016. "Views of Proof-Theoretic Semantics: Reified Proof-Theoretic Meanings." *Journal of Logic and Computation* 26 (2): 479–494.
- Francez, Nissim, and Roy Dyckhoff. 2010. "Proof-Theoretic Semantics for a Natural Language Fragment." *Linguistics and Philosophy* 33 (6): 447–477.
- Francez, Nissim, Roy Dyckhoff, and Gilad Ben-Avi. 2019. "Proof-Theoretic Semantics for Subsentential Phrases." *Studia Logica* 94 (3): 184–231.
- Friedman, Michael. 1974. "Explanation and Scientific Understanding." *The Journal of Philosophy* 71 (1): 5–19.
- . 2001. *Dynamics of Reason: The 1999 Kant Lectures at Stanford University*. Stanford: CSLI Publications.
- Gentzen, Gerhard. 1935a. "Untersuchungen über das Logische Schliessen I." *Mathematische Zeitschrift* 39 (2): 176–210.
- . 1935b. "Untersuchungen über das Logische Schliessen II." *Mathematische Zeitschrift* 39 (3): 405–431.
- Glazier, Martin. 2017. "Essentialist Explanation." *Philosophical Studies* 174 (11): 2871–2889.
- Gorman, Michael. 2005. "The Essential and the Accidental." *Ratio* 18 (3): 276–289.
- Hale, Bob. 1996. "Absolute Necessities." *Philosophical Perspectives* 10: 93–117.
- . 2013. *Necessary Beings: An Essay on Ontology, Modality, and the Relations Between Them*. New York: Oxford University Press.

- Hallnäs, Lars. 2006. "On the Proof-Theoretic Foundation of General Definition Theory." *Synthese* 148 (3): 589–602.
- Hallnäs, Lars, and Peter Schroeder-Heister. 1990. "A Proof-Theoretic Approach to Logic Programming I: Clauses as Rules." *Journal of Logic and Computation* 1 (2): 261–283.
- . 1991. "A Proof-Theoretic Approach to Logic Programming II: Programs as Definitions." *Journal of Logic and Computation* 1 (5): 635–660.
- Hempel, Carl. 1965. *Aspects of Scientific Explanation, and Other Essays in the Philosophy of Science*. New York: Free Press.
- Hjortland, Ole, and Shawn Standefer. 2018. "Inferentialism, Structure, and Conservativeness." In *From Rules to Meanings: New Essays on Inferentialism*, edited by Ondřej Berna, Vojtěch Kolman, and Ladislav Koreň, 115–140. New York: Routledge.
- Hofweber, Thomas. 2009. "Ambitious, Yet Modest, Metaphysics." In *Metametaphysics: New Essays in the Foundations of Ontology*, edited by David J. Chalmers, David Manley, and Ryan Wasserman, 260–289. New York: Oxford University Press.
- Jubien, Michael. 2009. *Possibility*. New York: Oxford University Press.
- Kant, Immanuel. 1998. *Critique of Pure Reason*. Edited and translated by Paul Guyer and Allen W. Wood. New York: Cambridge University Press.
- Khalifa, Kareem, Jared Millson, and Mark Risjord. 2018. "Inference, Explanation, and Asymmetry." *Synthese* 198: 929–953.
- Kitcher, Philip. 1989. "Explanatory Unification and the Causal Structure of the World." In *Scientific Explanation*, edited by Philip Kitcher and Wesley C. Salmon, 410–505. Minneapolis: University of Minnesota Press.
- Kneale, William, and Martha Kneale. 1962. *The Development of Logic*. New York: Oxford University Press.

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- Koslicki, Kathrin. 2012. "Essence, Necessity, and Explanation." In *Contemporary Aristotelian Metaphysics*, edited by Tuomas E. Tahko, 187–206. New York: Cambridge University Press.
- Kratzer, Angelika. 1977. "What 'Must' and 'Can' Must and Can Mean." *Linguistics and Philosophy* 1 (3): 337–355.
- Kripke, Saul. 1963. "Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculi." *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 9 (5–6): 67–96.
- . 1965. "Semantical Analysis of Modal Logic II: Non-Normal Modal Propositional Calculi." In *Symposium on the Theory of Models*, edited by J. W. Addison, Leon Henkin, and Alfred Tarski, 206–220. Amsterdam: North-Holland.
- Ladyman, James and Don Ross, with David Spurrett and John Collier. 2007. *Every Thing Must Go: Metaphysics Naturalized*. New York: Oxford University Press.
- Lange, Marc. 2009. *Laws and Lawmakers: Science, Metaphysics, and the Laws of Nature*. New York: Oxford University Press.
- . 2016. *Because Without Cause: Non-Causal Explanations in Science and Mathematics*. New York: Oxford University Press.
- Lipton, Peter. 2004. *Inference to the Best Explanation, Second Edition*. New York: Routledge.
- Litland, Jon Erling. 2015. "Grounding, Explanation, and the Limit of Internality." *Philosophical Review* 124 (4): 481–533.
- . 2016. "Pure Logic of Many-Many Ground." *Journal of Philosophical Logic* 45 (5): 531–577.
- . 2017. "Grounding Ground." In *Oxford Studies in Metaphysics*, edited by Karen Bennett and Dean W. Zimmerman, 10: 279–316. New York: Oxford University Press.

- Litland, Jon Erling. 2018. "Pure Logic of Iterated Full Ground." *Review of Symbolic Logic* 11 (3): 411–435.
- Martin-Löf, Per. 1996. "On the Meanings of the Logical Constants and Justifications of the Logical Laws." *Nordic Journal of Philosophical Logic* 1 (1): 11–60.
- Millson, Jared, Khareem Khalifa, and Mark Risjord. 2018. "Inferentialist-Expressivism for Explanatory Vocabulary." In *From Rules to Meanings: New Essays in Inferentialism*, edited by Ondřej Beran, Vojtěch Kolman, and Ladislav Koreň, 155–178. New York: Routledge.
- Millson, Jared, and Christian Straßer. 2019. "A Logic for Best Explanations." *Journal of Applied Non-Classical Logics* 29 (2): 124–128.
- Morvarid, Hashem. 2017. "Finean Essence, Logical Necessity, and Pure Logical Properties." *Synthese* 195 (11): 4997–5005.
- Nolan, Daniel. 2014. "Hyperintensional Metaphysics." *Philosophical Studies* 171 (1): 149–160.
- Norton, John D. 2003. "A Material Theory of Induction." *Philosophy of Science* 70: 647–670.
- . n.d. *The Material Theory of Induction*. http://www.pitt.edu/~jdnorton/homepage/cv.html#material_theory.
- Nutting, Eileen S., Ben Caplan, and Chris Tillman. 2017. "Constitutive Essence and Partial Grounding." *Inquiry* 61 (2): 137–161.
- Peacocke, Christopher. 1976. "An Appendix to David Wiggins' 'Note'." In *Truth and Meaning: Essays in Semantics*, edited by Gareth Evans and John McDowell, 313–324. New York: Oxford University Press.
- Peregrin, Jaroslav. 2007. "Extensional vs. Intensional Logic." In *Handbook of the Philosophy of Science*, edited by Dale Jacquette, vol. 5: Philosophy of Logic, 913–942. New York: Oxford University Press.
- . 2020. "Rudolf Carnap's Inferentialism." In *The Vienna Circle in Czechoslovakia*, edited by Radek Schuster, 97–109. Cham: Springer.

- Pezlar, Ivo. 2017. "Review of *Proof-Theoretic Semantics* by Nissim Francez." *Mind* 126 (501): 299–304.
- . 2018. "Proof-Theoretic Semantics and Hyperintensionality." *Logique et analyse* 61 (242): 151–161.
- Poggiolini, Francesca. 2016. "On Defining the Notion of Complete and Immediate Formal Grounding." *Synthese* 193 (10): 3147–3167.
- . 2018. "On Constructing a Logic for the Notion of Complete and Immediate Formal Grounding." *Synthese* 195 (3): 1231–1254.
- Prawitz, Dag. 1965. *Natural Deduction: A Proof-Theoretic Study*. Uppsala: Almqvist / Wiksell.
- . 1971. "Ideas and Results in Proof Theory." In *Proceedings of the 2nd Scandinavian Logic Symposium: Studies in Logic and the Foundations of Mathematics*, 63: 235–307. London: North-Holland.
- . 1973. "Towards a Foundation of a General Proof Theory." In *Logic, Methodology and Philosophy of Science IV*, edited by Patrick Suppes, Leon Henkin, Athanase Joja, and Grigore C. Moisil, 74: 225–250. Amsterdam: North-Holland.
- . 2006. "Meaning Approached Via Proofs." *Synthese* 148 (3): 507–524.
- Price, Huw. 2013. *Expressivism, Pragmatism and Representationalism*. New York: Cambridge University Press.
- Quine, Willard V. O. 1969. *Ontological Relativity and Other Essays*. New York: Columbia University Press.
- Rosen, Gideon. 2010. "Metaphysical Dependence: Grounding and Reduction." In *Modality: Metaphysics, Logic, and Epistemology*, edited by Bob Hale and Aviv Hoffman, 109–136. New York: Oxford University Press.
- Schroeder-Heister, Peter. 1984. "A Natural Extension of Natural Deduction." *Journal of Symbolic Logic* 49 (4): 1284–1300.

- Schroeder-Heister, Peter. 2008. "Proof-Theoretic Versus Model-Theoretic Consequence." In *The Logica Yearbook 2007*, edited by Michal Peliš, 187–200. Prague: Filosofia.
- Schurz, Gerhard. 1999. "Explanation as Unification." *Synthese* 120 (1): 95–114.
- Schurz, Gerhard, and Karel Lambert. 1994. "Outline of a Theory of Scientific Understanding." *Synthese* 101 (1): 65–120.
- Sedlar, Igor. 2009. "C. I. Lewis on Possible Worlds." *History and Philosophy of Logic* 30 (3): 283–291.
- Sellars, Wilfrid. 1958. "Counterfactuals, Dispositions, and the Causal Modalities." In *Minnesota Studies in the Philosophy of Science, Vol. II: Concepts, Theories, and the Mind-Body Problem*, edited by Herbert Feigl, Michael Scriven, and Grover Maxwell, 225–308. Minneapolis: University of Minnesota Press.
- . 2007. "Inference and Meaning." In *In the Space of Reasons: Selected Essays of Wilfrid Sellars*, edited by Kevin Scharp and Robert B. Brandom, 3–27. Cambridge: Harvard University Press.
- Skiles, Alexander. 2015. "Essence in Abundance." *Canadian Journal of Philosophy* 45 (1): 100–112.
- Stovall, Preston. 2015. "Chemicals, Organisms, and Persons: Modal Expressivism and a Descriptive Metaphysics of Kinds." PhD diss., University of Pittsburgh.
- . 2019. "Characterizing Generics are Material Inference Tickets: A Proof-Theoretic Analysis." *Inquiry* published online March 18, 2019: 1–37.
- . 2020. "Proof-Theoretic Semantics and the Interpretation of Atomic Sentences." In *The Logica Yearbook 2019*, edited by Igor Sedlár and Martin Blich, 163–178. Rickmansworth: College Publications.
- . Forthcoming A. *The Single-Minded Animal: Shared Intentionality, Normativity, and the Foundations of Discursive Cognition*. New York: Routledge.

- . Forthcoming B. "The Metaphysics of Practical Rationality: Intentional and Deontic Cognition." *Journal of the American Philosophical Association*.
- . Forthcoming C. "Modelling Descriptive and Deontic Cognition as Two Modes of Relation Between Mind and World." *Pacific Philosophical Quarterly*.
- Strawson, Peter F. 1959. *Individuals: An Essay in Descriptive Metaphysics*. Oxford: Routledge.
- Strevens, Michael. 2008. *Depth: An Account of Scientific Explanation*. Cambridge: Harvard University Press.
- Teitel, Trevor. 2019. "Contingent Existence and the Reduction of Modality to Essence." *Mind* 128 (509): 39–68.
- Thomasson, Amie. 2009. "Non-Descriptivism about Modality: A Brief History and Revival." *Baltic International Yearbook of Cognition, Logic and Communication* 4 (1): 1–26.
- . 2016. "Metaphysical Disputes and Metalinguistic Negotiation." *Analytic Philosophy* 57 (4): 1–28.
- Tillman, Chris. 2016. "Essence Facts and Explanation." *Thought* 5 (3): 190–195.
- Tuboly, Ádam Tamas. 2015. "Quine and Quantified Modal Logic: Against the Received View." *Organon F* 22 (4): 518–545.
- . 2017. "From 'Syntax' to 'Semantik': Carnap's Inferentialism and Its Prospects." *Polish Journal of Philosophy* 11 (1): 57–78.
- . 2018. "The Early Formation of Modal Logic and Its Significance: A Historical Note on Quine, Carnap, and a Bit of Church." *History and Philosophy of Logic* 39 (3): 289–304.
- Więckowski, Bartosz. 2011. "Rules for Subatomic Derivation." *Review of symbolic logic* 4 (2): 219–236.

- Wiggins, David. 1976. "The *De Re* 'Must'." In *Truth and Meaning: Essays in Semantics*, edited by Gareth Evans and John McDowell, 285–312. New York: Oxford University Press.
- Wildman, Nathan. 2013. "Modality, Sparsity, and Essence." *The Philosophical Quarterly* 63 (253): 760–782.
- . 2016. "How (Not) to Be a Modalist About Essence." In *Reality Making*, edited by Mark Jago, 177–195. New York: Oxford University Press.
- . 2018. "Against the Reduction of Modality to Essence." *Synthese*.
- Williamson, Timothy. 2007. *The Philosophy of Philosophy*. Malden: Blackwell Publishing.
- Wilson, Mark. 2006. *Wandering Significance: An Essay on Conceptual Behavior*. New York: Oxford University Press.
- Woodward, James. 2003. *Making Things Happen: A Theory of Causal Explanation*. New York: Oxford University Press.
- Zalta, Edward N. 2006. "Essence and Modality." *Mind* 115 (459): 659–693.
- Zylstra, Justin. 2019. "Essence, Necessity, and Definition." *Philosophical Studies* 176 (2): 339–350.