COMPUTER ASSISTED ANALYSIS OF TONAL STRUCTURE

IN THE CLASSICAL SYMPHONY

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The research of which this paper is part the outcome was stimulated by two observations concerning the way in which tonal structure is discussed in twentieth century considerations of the classical symphony. Firstly, that the specific examples discussed are frequently presented as exceptional; and secondly, that there is much talk of 'distant keys' but few suggestions as to how one might measure tonal 'distance'. Two questions arise, with so many exceptions, where is the evidence for the norms, and, is there a statistically sound basis for the generalisations being made, whether explicitly or implicitly?

What follows is an attempt at resolving both issues, firstly by establishing an appropriate eighteenth century model of key relationship and then by applying that model consistently to a large body of related works for computer-assisted statistical analysis, thus permitting the quantification of both tonal distance and the norms of tonal structure in the works studied.

Many twentieth century commentators, not least among them Tovey, have hailed the importance of tonal architecture in classical music. This was an essential shift in emphasis after the predominance of largely thematic analyses in the nineteenth century. Only recently, however, does there seem to have been a revival of interest in what composers of the eighteenth century themselves had to say about key structure in the music of their own time.
James Grassineau, Gentleman, in his Musical Dictionary of 1760, under the entry KEY, observes that,

'as in action there is a subject, viz. some principal person or thing, to which the discourse is refer'd, and which is always kept in view, that nothing unnatural or foreign to the subject may be brought in: so in every regular piece of music, there is one sound, viz. the Key, which regulates all the rest; the piece begins and ends in this; and this is as it were the musical subject, to which a regard must be had in all the other sounds of the piece.

Again, as in action there are several distinct articles which refer to different subjects, yet so as they may have a visible connection with the principal subject, which regulates and influences the whole; so in music, there may be various subaltern subjects, that is, various Keys, to which the different parts of the piece may belong; but then they must be all under the influence of the first and principal Key, and have a connection with it.\(^1\)

Under MODULATION, Grassineau stresses the twin needs of tonal variety and tonal unity, the former being satisfied by modulation, the latter by the choice of those keys into which 'the harmony may be conducted with propriety.'\(^2\) Grassineau fails to identify these keys at all precisely but the German theorists Werckmeister, Helmholtz, Mathias\(^3\), C.P.E. Bach and J.J. Quantz all discuss the subject obliquely or directly and provide valuable insights into eighteenth century conceptions of key relationship.

All, in fact, seem to consider key choice of paramount importance both within and between movements and certainly of equal significance with thematic structure.
Hainichen and C.P.E. Bach give us two of our clearest pictures of an eighteenth century model of key relationships. Hainichen in his Neun erfundene und gedichtete Anweisung ... des General Baues, (Hamburg, 1711) and Bach in his Versuch ... (Hamburg, 1753). Chapter seven of the Bach treatise is entitled 'Improvisation' and deals at length with questions of key relationship. Most importantly, it identifies the two primary relationships as being that between tonic and dominant and that between the tonic and its relative. Bach is at pains to point out the symmetry of the system, equating the pairs C major - G major and C major - A minor with the pairs A minor - E minor and A minor - C major, respectively. He clearly recognizes these two primary relationships as being the basis of the circle of fifths, which is generated by their systematic repetition and which he invokes in justification of his categorization of keys into 'closely related', 'remote' and 'most distant'. Keys 'stand in varying distances from the tonal center (as) may be seen from an examination of the well-known Circle of Keys.\(^4\)

Meirich Schenker took particular interest in this chapter of Bach's and in part bases his system of tonal analysis upon similar premises to those of Bach. Another scholar to recognize the usefulness of the circle of fifths in measuring tonal distances is Laszlo Somfai\(^5\) although he takes the method from a nineteenth century source: A. von Oettlen's Harmonielehre im dueter Entwickelung (1866).

In any event, the measurement of tonal distance employed in the present study resembles closely that expounded by Somfai, is justified in terms of eighteenth century music theory, and is based on the key circle illustrated below\(^6\) (Fig. 1).
The importance of C.P.E. Bach in this question cannot be over-stressed in view of Haydn's reported admiration for both the man and the work and Bach's appeal that Haydn's works showed him to be a true disciple. In turn, Haydn's significance as a classical symphonist scarcely needs reiteration.

The intention in seeking a method for measuring tonal distance was to make possible the treatment of key as a continuous variable, in fact to make it possible to graph tonality from measure to measure such as one can graph temperature from hour to hour or minute to minute. A simple two-dimensional graph with key forming the vertical axis and measure number the horizontal axis allows one to preserve ordinal, relational and proportional information.
Under this system of graphical representation, tonal distance is defined in terms of distance from the tonic around the circle of keys, with sharper keys having positive values and flatter keys negative ones. Each graph is centered on the tonic key for the movement and the difference in mode between a major key and its relative minor is represented by a difference in plotted character: a small square for major mode, an asterisk for minor mode, (Fig. 2).

![Graph showing tonal distance and mode difference]

Fig. 2.

The vertical axis consists of two parallel lists of keys, the major keys on the left with their relative minors to the right. Repeats and double bar lines are also indicated.
It will be observed that space is provided for intermediate levels between keys. This is to allow for those measures in which, while the music is clearly in the region of a given tonal center, that center either has not yet been established or some passing harmonic manoeuvre has disturbed its 'established' status. A typical structural feature requiring this intermediate status is the dominant preparation, for which it would be wrong to symbolise the target key as being established while the music is clearly hovering, nonetheless, 'in and around' it. Since the 'climbing' manoeuvre which culminates dominant preparations and confirms the establishment of a new key is frequently a perfect cadence in that key, it would seem appropriate to place this intermediate level on the positive, that is, dominant, side of the established level. Thus we have for each key two levels: the tonic or 'established' level and the dominant or 'in and around' level. For instance, in Fig. 2, above, measures 22 through 26 have a tonal value of 1.5, since they are on the dominant level of B flat major which in turn is one step around the circle of fifths to the sharp side of the tonic, E flat major.

Similarly, rapidly modulating passages can be given a series of tonal values by recognising the 'instantaneous key' apparent at the end of each measure. An analogous procedure is the calculation of the instantaneous velocity of an accelerating body. The tangent to the curve representing displacement vs. time is calculated for the given time, and the angle of this tangent gives the velocity the particle would have were the accelerating force to cease at that point: an time. (Fig. 3).
In like manner the tonal analyst estimates the nearest key center at the end of a measure by answering the question, "What key would I be left in were the music to stop at this point?"

This notion of tonal inertia is also invoked in dealing with that charm-ingly Haydnque event, the general pause. Whatever tonal state (often a slightly disturbed one) preceded the pause is deemed to continue through the brief silence.

The first application of this graphical method of analysis and representation was to Beethoven’s String Quartet in C sharp minor, Op. 131. The opening movement of this work is a cases of startling economy in its tonal plan. One hundred and twelve-verse measures long, it establishes a tonal level two fifths to the sharp side in the first sixty measures and crosses then to a point two fifths flatter than the tonic, finally.
returning to the home key in a manner that completes a superbly balanced "moonmail of key shifts and temporal proportions.

A quick comparison with a recent study in the same key written by J.J. Bach within exactly one hundred years previously revealed an apparent far less architectural approach to tonal design. This was due to the fact that in the intervening century one might expect to detect a much higher number of works exhibiting a tendency to balance time spent in keys sharper than the tonic with a similar time spent in flatter keys, and, in the case of sonata form movements, this flattening would be most noticeable in development sections.

These are essentially statistical propositions: computational assistance was clearly indicated. A most suitable body of works for examination was also quick to recommend itself: the entire symphonic output of J. Haydn. Produced with a fair degree of regularity over nearly forty years and in large enough numbers to warrant some confidence in the application of statistical techniques as a solid foundation for attempting to generalize about classical symphonic tonal structure, the Haydn symphonies are ideally suited to the purpose.

The initial tonal analysis was performed at the keyboard, sight-reading, to ensure that as well as notational evidence contributed to the estimated 10,000 analytical decisions that were made. Once the score had been annotated to indicate changes of key, the information was encoded (Fig. 4).
These data then passed through two editing programs which checked for punching and logical errors. When these had been corrected, two programs (MATH and MICRO) produced graphical representations of the analyses. The first simply printed a straight-line graph of each movement's tonal structure, centered on the tonic and continuing an successive peaks as required for longer movements. The second graphing program converted all movements to a form one hundred plotting characters long by a process of interpolation. Some movements were stretched and some reduced in length, but temporal proportion was always maintained, thus permitting easy direct visual comparison of any total curve with any other.

In addition, a number of statistics was calculated for each movement, the most important for our current purposes being mean tonic level, standard deviation and the sharpest and flattest levels reached. At present most of the information is lost in the interpolation procedure. In the future it is hoped to rectify this by the use of a plotter and colour-coding note difference. Both graphical representations of the first movement of Haydn's Symphony No. 96 appear below. (Fig. 5)
Fig. 5a. Csound output.
Further progress in such tasks as plotting other statistics against
axial number (as approximation to compositional order based on R.C.,
Robins-Ludlow's chronology), superimposing standardized graphs, and
finding the mean tonal curves of whole categories of movements. Those
latter are also able to examine such things as the range of structural
variation within the category and the main tonal tendencies at various
proportional points along those curves.

The results of the Haydn study were gratifyingly conclusive. Taking first
the question of regular tonal tendencies during development sections, of
the 140 movements which could be identified as being examples of sonata
form, 5.7% exhibited no flattening whatever during the development section.
19.9% exhibited some, but for less than 2% of its duration, 77.2% dealt in
flat keys relative to the tonic between 25 and 50% of the time, and 77.7%
travelled well to the flat side for over half of the development's length.
Thus 94.3% display at least some tendency of this sort and 74.8% exhibit a
significant degree of much flattening.

Beautiful sinuosities occur even amongst the earliest symphonies and may be
found at all stages of Haydn's career, although somewhat less frequently
during the middle years. Figure 2, above, exemplifies the early sinuosity,
while Figure 6a, below, is an interesting expression of C.P.E. Bach's
symmetrical conception of key relationships being applied by Haydn. In a
minor key, it proceeds to the dominant minor during the exposition, to
flat major keys for the development and returns to the tonic (minor) for the
entire recapitulation. The second movement of Symphony No. 96 (Fig. 6b)
and No. 96, I (Fig. 5b) are middle and late examples, respectively, of the
same elegant design.

\[ \text{Fig. 6a.} \]
The ubiquity of this design can be seen in the next final outlines (Fig. 7a,b).
It is worth noting that the degree of flattening in the late middle movements is considerably more pronounced than that in the earliest ones.

Does this common flattening in the development section, then, produce a tonally balanced form? A measure of balance is a simple mean of all tonal values for the movement. Looking at these mean tonal values throughout Haydn’s symphonic career, one can detect a definite trend downwards, with first movements, on average, coming nearest to exact balance in the early seventies and late eighties. The final tipping of the balance in favour of flat keys only occurs in the London symphonies, while middle movements and final movements display this degree of flattening in the seventies. (Fig. 8).
The cause of this gradual flattening in the overall balance can be deduced from the gradual expansion in the repertoire of keys Haydn was prepared to explore within the bounds of a single movement. (Fig. 9).

Fig. 9.
In particular it should be noted that the outward expansion is more pronounced and consistent on the flat side. That the amount of time spent in distant keys was not insignificant can be seen from the upward trend in the mean standard deviations. (Fig. 10).

Apart from this clear-cut confirmation of a tendency in Haydn's symphonic sonata forms to balance time spent in sharp regions with flat development sections, a number of other results have been obtained from this research. The graphical representation of tonal structure has certainly proved useful as a pedagogical tool in the teaching of structural tonal concepts in undergraduate music history and theory courses.
A less expected line of development that produced spectacular results was a study of temporal proportions spliced off by reading Eilson Camp's dissertation on temporal proportion in Mozart's keyboard works. Like Mozart, Beethoven displays a strong predilection for the Golden Ratio, seems to have some liking for the ratio 2:1 but, unlike Mozart, no interest in the ratio 7:12 (Fig. 11a,b).
At first the possibility of using aspects of tonal structure to help identify compositors seemed remote, however. Continuing research into the symphonies of Mozart, Beethoven, C.P.E. Bach and Sammartini suggests that tonal architecture can be quite idiosyncratic and could possibly aid in ascription disputa; given large enough numbers of authenticated works for statistically meaningful comparisons to be made. Mozart, for instance, visits flat-side keys in his early symphonies noticeably less often than Haydn does. C.P.E. Bach is different from all the others in his nervous habit of langthly delaying key-establishment, resulting in a characteristic histogram of percentage incidence of key lewises.

Clearly this analytical and computational system has room for improvement and expansion: the use of a printer instead of a printer for GRAPHEL, the production of transition matrices to investigate more deeply questions of key sequences, the automation of tonal analysis using Gabura's algorithm, but with direct piano-keyboard interfacing, and the ultimate development of parallel analytical techniques for the systematic study of other musical parameters all call for further time and effort. In view of the usefulness of the results obtained to date, however, such effort will not be wasted.

ps2

2. Ibid. p.118.

3. One of the most recent and most useful discussions of this topic is Joel Lester's 'The recognition of major and minor keys in German theory: 1680 - 1730' in the *Journal of music theory*, Spring 1978, vol. 22 no. 1, pp. 65 - 103.

4. Bach, C.P.E. *Essay on the true art of playing keyboard instruments*, trans. and ed. William J. Mitchell, Cassell, London, 1949, pp. 434-5. Mitchell considers this to be a reference to the Heinichen circle but Bach's discussion of both the primary relationships and key symmetry points to the familiar pair of concentric major and minor key circles. It is hoped that further research will permit a firmer identification of the specific circle to which Bach is referring. It is clear, even from Mitchell, that the concentric model was known to Kircher in the late seventeenth century.


7. A number of alternative methods of representing tonal structure have been examined including the simple list of keys and measure numbers, Schenkerian analysis and Graham George's graphical representation. None of them preserves all three types of information and none presents them in a readily available form visually. Schenker's is the only method, for example, which gives relational information and that in a form which makes the assigning of tonal values in the arithmetic sense difficult.

8. This is not to say that J.S. Bach never produced works of comparable interest in tonal design: the comparison between the Bach and Beethoven opus 88a simply the immediate stimulus for a more far-reaching set of comparisons.

9. The text used throughout was H.C. Robbins Landon's excellent edition of the complete symphonies.
