COMPOSING WITH SIEVES:

STRUCTURE AND INDETERMINACY IN-TIME.

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ABSTRACT

Introduced by Xenakis 50 years ago, sieves have proved to be a relevant and robust device for music composition. Examples of complex and symmetric sieves usage in original works are presented along with a few possible applications not explored before. The dichotomy between predetermined abstract structures such as sieves and their actualization through random procedures is discussed and it is also shown that in the hands of an innovative musician, sieves not only serve the craft aspect of composing but could also reveal as well as impact deeper levels of thinking.

1. PERIODICITY, WEIGHTS, AND SYMMETRY.

Sieves were introduced during the early 1960s by Xenakis in his works but remained a rather esoteric topic until rather recently when a number of writings on the subject have appeared - Ariza [1], Gibson [6], Exarchos and Jones [5], Solomos [7] to name some- testifying to the relevance and importance of this device for music composition. Since many basic aspects and in particular sieve analysis and construction have been discussed previously, only a few brief reminders are necessary here.

Sieves are logical filters expressed as boolean operations on congruence modulo classes. A trivial case is that of a sieve containing equivalence classes denoted by various indices (following the notation used by Xenakis) of a single modulo:

\[ \mathbb{Z}_p \setminus \mathbb{Z}_{13} \cup \mathbb{Z}_3 \cup \mathbb{Z}_1 \]

This formula will generate a periodic sequence of numbers with 13 the only modulo able to define elements of the sieve since it is a prime number. Messiaen's modes with limited transpositions can be generated with simple periodic sieves:

\[ 3_3 \cup 3_1 (\text{second mode, an octavating scale}) \]

while the expression offered by Xenakis in Formalized Music [13] for generating the major scale contains two modulo terms and a more involved set of operations:

\[ (4_2 \cup 3_3)(4_3 \cup 4_4)(3_3 \cup 3_4)(4_3 \cup 4_4)(3_3 \cup 4_3) \]

Figure 1. Modulo weights

The extreme ones would favor the tonic triad {0, 4, 7} over the dominant triad {5, 11}. Applied to diatonic pitches, the results in this example would favor the tonic triad (0, 4, 7) over the dominant tritone (5, 11).

Sieves that produce symmetric intervals between numbers contain modulo terms that have symmetric indices. A simple nonretraversable rhythm:
attacks durations
0 1 9 11 19 20
1 8 2 8 1

Figure 2. Nonretrogadable rhythm
will be produced by the sieve:
\[(5_4 \cap 4_4) \cup (3_4 \cup 5_4) \cap (4_4 \cup 4_4)\]
where the terms 5_4 and 4_4, are symmetric with respect to the origins 5 and 4 since the sieve is periodic and 5_4 = 5_4. The sieve generating the Dorian mode shows a similar balance:
\[(3_4 \cap 4_4) \cup (4_4 \cap 4_4) \cup (3_4 \cap 4_4) \cup (4_4 \cap 4_4)\]

Figure 3. Rhythmic palindrome ostinato

The nonretrogadable rhythm forms a firm and elaborate scaffolding for the piece but its length and complexity make it difficult to be detected by the listener. In order to create a more versatile relationship for the piece than the nonretrogadable rhythm, the complexity of the pitch material, the dynamic levels and other parameter values are incremented constantly as we advance from left to right, along with the general entropy - the measure of disorder, that defines the passing of time.

An interesting case is that of rhythmic sieves that are symmetric and extended over very large areas of a piece; they can be used to create structures similar to that of the first movement of the Symphony Op. 21 by Webern or Machaut's Ma fin est mon commencement.

Multiple-entry sieves, an even more elaborate construct, could be described as involving conditional probability, or as dimensional matrices (not unlike the sequence of "screens" used by Xenakis to generate Analytique A et B) or as related to the more common sieves through the use of equivalence modulo relations of the type:
\[k_1 m_1 + k_2 m_2 + \ldots + k_n m_n + n\]

This expression is helpful, for instance, when determining the position of an attack measured in the thirty minute duration of one of the performances of the Dorian mode. It must be clear by now that sieves with hundreds of elements, or as multidimensional matrices (not unlike the sequence of "screens" used by Xenakis to generate Analytique A et B) or as related to the more common sieves through the use of equivalence modulo relations of the type:
\[k_1 m_1 + k_2 m_2 + \ldots + k_n m_n + n\]

Such an obvious way to apply a sieve is to use all its elements in the same way every note of a scale is employed in a traditional piece; assigning weights to its components will enhance its internal organization without changing the way it functions. In the works mentioned above as well as in many other ones, sieves were treated as templates, as potentialities that might or might not be fully realized. In the time domain, for instance, a weighted sieve engendering a large number of attacks defines only places where sounds could occur without totally guaranteeing that any would actually be heard at any particular location. In a computer-assisted composition, a linear density smaller than the density of the sieve coupled with selections based on random procedures will insure that not all possible locations of the piece - a pitch sieve rich in available choices might not be fully utilized at the local level thus masking the extent of the pitch reservoir available for a larger area.

The play between structure and randomness, between determism (sieve) and chance, mirrors the natural world where the laws of physics allow for more virtual events than actual happenings.

The predetermined structure (physical laws, sieve, etc.) cannot be contradicted and its actualization has to conform. At the same time, these actual manifestations are unpredictable; the result of causal chains too complex to follow, of pure chance or, in our case, the result of applying stochastic distributions. This way, the music results from the meeting of the possible ascertained by the sieve and the probable represented by random procedures.

There are many flowers
But few will bear fruit;
They all knock at life’s gate
Yet many blossoms die.

Such ideas are implemented through DISSCO when producing manifold compositions, multiple variants of the same piece that emphasize the interaction between structure and indeterminacy [11]. They are composition classes generated by a computer that runs a program containing elements of indeterminacy and reads the same data for each variant. As members of an equivalence class, they share the same structure and are the result of the same processes, but deviate in many specific ways each time specific events are arranged in time. Similar to faces in a crowd, they have the same basic features but differ in details and reflect various personalities.

4. ABOUT TIME.

Another way of looking at the dichotomy between structure and indeterminacy is the excelling use of the outside-time/in-time categories considered by Xenakis. Sieves are typical examples of outside-time abstract structures, preceding any attempts to fashion specific events, while the in-time product, the piece as a whole, is a variant of the outside-time domain in DISSCO’s case, is the result of random processes being applied to abstract templates. Such outside-time “architectures” (as Xenakis calls them) are "outside" in time and betray a modernist-structuralist way of

123
attacks
0 1 9 11 19 20
1 8 2 8 1

Figure 2. Nonretrogadable rhythm

\[(S_5 \cap U_4) \cap (S_5 \cap U_5) \cap (U_4 \cap U_5)\]

where the terms \(S_5\), \(S_4\) and \(U_4\), \(U_5\) are symmetric with respect to the origins \(S_5\) and \(S_4\) since the sieve is periodic and \(S_5 = S_4\). The sieve generating the Dorian mode shows a similar balance:

\[(S_4 \cap U_4) \cap (S_4 \cap U_5) \cap (S_3 \cap U_4) \cap (U_4 \cap U_5)\]

An interesting case is that of rhythmic sieves that are symmetric and extended over very large areas of a piece; they can be used to create structures similar to that of the first movement of the Symphonic Op. 21 by Webern or Machaut's Non finitum est mon commencement.

Multiple-entry sieves, an even more elaborate construct, could be described as involving conditional probability, or as multidimensional matrices (not unlike the sequence of "screens" used by Xenakis to generate Analogik A et B) or as related to the more common sieves through the use of equivalence modulo relations of the type

\[k_1 m_1 + k_2 m_2 + \ldots + k_n m_n = n\]

This expression is helpful, for instance, when determining the position of an attack measured in the smallest time quanta available. With \(k_2\) being the number measure, \(k_1\) the number of a beat in a 3/4 measure and the sixteen the shortest duration item or the EDU (Elementary Displacement Unit in the terminology introduced by Xenakis),

\[m_1 = 4\text{ sixteenths},
\]

\[m_2 = 3\text{ beats} - m_2 = 12,\]

\[m_3 = 4\text{ sixteenths} + 3\text{ beats} + m_3 = 23,\]

\[m_4 = 4\text{ sixteenths} + 1\text{ beat} + m_4 = 24,\]

\[m_5 = 4\text{ sixteenths} + 2\text{ beats} + m_5 = 26,\]

\[m_6 = 4\text{ sixteenths} + 3\text{ beats} + m_6 = 27\]

(first member of each group is always 0). If instead we consider \(m_1\) the number of all dynamic levels in a piece, \(m_2\) the number of all available pitches + \(m_2\), \(m_3\) a particular pitch level, \(k_1\) a particular pitch, and \(k_2\) a particular instrument in a group of size \(k_1\), we can create orchestration constraints. The period of such a sieve will vary with the chosen module numbers but will have to be divisible of \(m_1\) total number of instruments.

2. APPLICATIONS

A symmetric rhythmic sieve that includes over 100 modulo terms and operators was used to create an extended palindrome in Cuniculi, for five tubas [10]. The same symmetric sieve is repeated throughout the piece in an estimation of palindromes that cover its entire 10:30 minutes duration:

\[\text{etc.} \]

Figure 3. Rhythmic palindrome ostinato

The nonretrogadable rhythm forms a firm and elaborate scaffolding for the piece but its length and complexity make it difficult to be detected by the listener. In order to make that which will be, possible combinations for the \(\alpha\) portion of the sieve increases from 4 (for one iteration) to 9 (for two iterations) or 16 (for three iterations).

Next, probabilities are assigned to each of those pairs and, in the case of synthesized sounds, choices are made with the help of a random number generator. The stream of acoustic sounds is computed in advance and a new variant of the sieve is generated for each performance. For the violin, all alternatives are provided in the score and the performer is asked to choose one of them: the ostinato of rhythmic palindromes provides a foundation for areas of aleatory music while, at the same time, insuring the coherence and the integrity of the process. The live musician could choose what path to follow either ahead of time or, preferably, at the time of the performance. In the latter case, the choices made by the human artist on the spot will be influenced - if only subliminally - by the music coming out of the loudspeakers which, again, will be different every time.

Another rather straightforward approach of sieves consists in establishing pitch areas easily identifiable and related to each other. Here, the strategy defines the possible "modulating" from one area to the next. This can be done by selecting subsets of a sieve with many elements and either starting with smaller subsets and progressively adding new sounds or by using intersections between subsets, in other words manipulating the module terms of the more complex sieve. Admittedly, this is not a procedure as elegant and sophisticated as the metabolae proposed by Xenakis [13] but it is effective nevertheless.

All procedures mentioned above represent features available in DISSCO and have been employed in actual compositions. A number of other sieve applications are candidates for becoming future components of the software. Weighted sieves could control the presence and the amplitude of sound particles in additive synthesis either by creating models of acoustic instrumental sounds or, in a more abstract way, by paralleling other proportions in the structure of the work, the resonance of instrument bodies could be simulated through complex weighted sieves; and formants, a combination of both resonance and control of partials, could be created. Since in DISSCO, analogous operations are present at all structural levels, one can also conceive formants at the macro level of the piece as envisaged by Stockhausen or even as means of organizing the form of the piece as proposed by Boulanger (Third Piano Sonata).

3. TEMPLETLES AND REALIZATIONS

In the hands of a diligent and creative composer, sieves could become a powerful tool. Their usefulness can go beyond the craft aspect of composing and both reveal as well as impact deeper levels of thinking.

An obvious way to apply a sieve is to use all its elements in the same way every note of a scale is employed in a traditional piece; assigning weights to its components will enhance its internal organization without changing the way it functions. In the works mentioned above as well as in many other ones, sieves were treated as templates, as potentials that might or might not be fully realized. In the time domain, for instance, a weighted sieve engendering a large number of attacks defines only places where sounds could occur without guaranteeing that any would actually be produced at any particular location. In a computer-assisted composition, a linear density smaller than the density of the sieve coupled with selections based on random procedures will insure that not all possible outcomes will be realized. Similarly, a pitch sieve rich in available choices might not be fully utilized at the local level thus masking the extent of the pitch reservoir available for a larger area.

The play between structure and randomness, between determinism (sieve) and chance, mirrors the natural world where the laws of physics allow for more virtual events than actual happenings.

The predetermined structure (physical laws, sieve, etc.) can not be contradicted and its actualization has to conform. At the same time, these actual manifestations are uncertain events that are unpredictable, the result of casual chains, the play between structure and randomness, between the predetermined structure (physical laws, sieve, etc.) and its actualization yet many blossoms die. [4]

Such ideas are implemented through DISSCO when producing manifold compositions, multiple variants of the same piece that emphasize the interaction between structure and indeterminacy [11]. They are composition classes generated by a computer that runs a program containing elements of indeterminacy and reads the same data for each variant. As members of an equivalence class, they share the same structure and are the result of the same processes, but deviate in specific events that are arranged in time. Similar to faces in a crowd, they have the same basic features but differ in details and reflect various personalities.

4. ABOUT TIME.

Another way of looking at the dichotomy between structure and indeterminacy is the division of a musical piece into in-time/in-time categories considered by Xenakis. Sieves are typical examples of outside-time abstract structures, preceding any attempts to fashion specific events, while the in-time product, the piece as a whole, is a variant of a structure. In DISSCO’s case, it is the result of random processes being applied to abstract templates. Such outside-time “architectures” (as Xenakis calls through the outside-in time and betrays a modernist-structuralist way of
thinking in the view of Jean-Jacques Nattiez [8] who points out that, by contrast, post-modernism is more concerned with the passing of time.

In fact, Xenakis also defines a third category, the temporal which might be puzzling until one realizes that although sound parameters have an Abelian (commutative) group structure for all possible time, being irreversible, does not have a group structure (i.e. there are no inverses). The temporal category underlines this peculiarity of the experience that it is apart from the other aspects of a musical composition. Now, although an asymmetric succession of arbitrary durations does not form a group, nontetragonal rhythms and iterations such as ostinato or traditional, constant steady meter, do. They create a cyclical time associated with the “eternal return” identified by Mircea Eliade as characteristic to all rituals [3]; they compel us to participate in an exemplary action that is repeated at regular intervals and supersede our perception of the mundane flowing of “ordinary” time.

In DISSCO, indeterminate processes that distribute events in-time complement or even upset these unihistorical, cyclical returns created by sieves templates. Since a variant of the manifold is generated in one seamless run, it owes its distinct integrity, its own “personality”, to a particular “history” provided by a sequence of random numbers. However, a random number generator actually produces pseudo random numbers and the same seed engenders the same exact output. An interesting situation is then created: the events in the piece depend on indeterminacy but the sequence of random numbers, the source of randomness, is itself a deterministic chain. One more layer in the play of randomness in-time is a pseudo random process which is itself a deterministic chain, and shared, and on the way storytellers typically use teller as little as possible from moving about and using their arms and hands to gesture when not using the instrument, and to some extent even when the instrument is being used. We also did not want the instrument to require visual attention. These criteria determined our choice of a hand-held mobile device as the instrument.

The fact that they system can control any sound, and any number of sounds, creates a challenge for interface design. Sounds must be selected and controlled in a manageable way for a storyteller whose primary goal is the story, not the instrument.

Another design goal for sonicBard is to maintain the fluid way that stories are shared among storytellers in oral traditions. This is a very difficult criterion for computer-based technology to meet, since typically physical media or installation on a device would be required. A story on the other hand is essentially shared once it has been told and heard. This design goal drove the entire project to be developed on the web platform. With sonicBard, “having” the sonically-enabled story after hearing it is as simple as pointing a mobile device browser at a URL.

### 2. SYSTEM DESIGN

The architecture for sonicBard consists of:
1) a collection of “sound models”;
2) a scene navigation and interface controller;
3) a system for organizing collections of sounds into “scenes” and providing mappings between the controller and the sounds.

#### 2.1. Sound Models

The collection of sound models are built using the formative W3C Web Audio API[5] (discussed below). In addition, we have constructed a library called jsaSound which provides utilities (opcode-like generators, unit converters, etc), templates (code for frequently used model structures), and of primary importance for the sonicBard project, a simple and consistent interface for all sounds.

The jsaSound interface makes a clear distinction between sound developers and sound users. Sound developers use JavaScript, the Web Audio API, and the jsaSound library tools to build sound models, whereas sound users such as game, music application, or web developers simply load, play, and set up relationships between application or user actions and interactive sound behavior. The API covering the key functionality of jsaSound models is shown in Table 1.

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**REFERENCES**