Proceedings of The 1980 International
Computer Music Conference

COMPOSING FROM A GEOMETRIC MODEL: FIVE-LEAF ROSE

by

Gary S. Kendall
Northwestern University

When one considers the vast range of possible applications of mathematics to music, composing from a geometric model would appear to be one of the simpler things to do. Certainly the geometry I have used in composing Five-Leaf Rose is conceptually simpler than set theory or Stochastic processes. The purpose of using any such method, of course, is to effect structural integration and unity. What gives this geometric approach useful musical properties is the depth to which it can be applied in a computer-generated composition. The unity is not achieved merely by the idea of a geometric figure, but by having a coordinated pattern of control on many different aspects of the composition. Nearly everything in Five-Leaf Rose, from the formal structure to the acoustic details is tied to this single model. The one exception, melody, is freely composed against the background of these highly organized elements. The geometric figure as such may not be intelligible to the audience, but the musical relationships derived from the figure are.

This method of composing might have been conceivable without the use of a digital computer, but would not likely have been carried very far without it. The computer removes all of the practical obstacles to this kind of composition and makes the "conceivable" also possible.

Proceedings of The 1980 International Computer Music Conference

The score to Five-Leaf Rose was prepared by a compositional program that computed the relationship of all of the acoustic details to the large-scale design, a task that would have been extremely unwieldy to do by hand. And, of course, it is only possible to have this kind of control on acoustic details with computer-generated sound.

Form and Design

The geometric figure of a five-leaf rose has five loops that intersect at a common point as shown in Fig. 1a. It can easily be drawn with one continuous line looping out and back in, out and back in, until there are five loops and the line returns to the first leaf. There are a number of properties of the five-leaf rose which underlie its use in the design of this composition. One property of the figure is that it is a continuous curve. This property guarantees that the composition will go through slow continuous changes (rather than having sudden dramatic contrasts). Another property is that the figure has an underlying pattern and this insures that the changes in the composition will repeat a pattern. Still another property is that, even though the curve itself is continuous, the angle it traces in relation to the origin has discontinuities which can serve as points of articulation in the composition. As shown in Fig. 1b, as one traces the first leaf beginning with the upper portion, the angle θ moves from 72° to 0° . When one enters the second leaf, the angle jumps to 144° and moves back toward 72° as one traces through the second leaf. This process repeats

Fig. 1a

Fig. 1b

every 72° until the entire figure is traced. In addition to the discontinuities in the angle, if one imagines that the curve is traced with a pen moving at a constant velocity, the rate of angular change varies and thus provides a large-scale pattern for rates of change that is linked to the shape of each leaf. The rate of angular change is high when tracing the most remote part of a leaf, because the curve is cutting directly across the angle. The rate of angular change is low when tracing the part of a leaf near the origin because the curve slopes away gently.

The geometric properties are given musical meaning by assigning musical parameters to the different dimensions of the graph. In the case of this figure, parameters are assigned according to polar coordinates. The angle θ controls all pitch and timbral aspects of the composition. For example, θ represents a timbral spectrum; all of the possible timbres in the composition are arranged in a circle. If one moved directly around the plane in a circle, the timbres would slowly and continuously modulate back and forth between string-like and wind-like, bright and dark, etc. (exactly how this is done will be explained in connection with the orchestra program).

The two-dimensional plane itself is also taken as a literal map of a physical plane on which the composition takes place. Each individual note has a unique position on the curve and thus a unique timbre. The notes are positioned on the average of every two degrees, and therefore, the density of the notes is greatest in the most remote part

Proceedings of The 1980 International Computer Music Conference

of the leaves (Fig. 2a). The form of the composition unfolds, not as the sound sources move, but as the audience location moves. It is as if the audience was on a boat tracing the curve of the five-leaf rose and as the boat moves, there are notes sounding ahead and behind it. (Fig. 2b). The speed of the audience changes in such a way that it is fastest as it passes through the origin and it is slowest at the tip of each leaf, somewhat in the manner of a planetary orbit.

One of the consequences of the pitch-timbre scheme is that the composition is based on a single stream of pitches that unfolds as the audience moves around the figure. The notes actually sounding though are made to run slightly ahead of and behind the audience's location. There are six voices sounding simultaneously throughout. Whenever the audience's location passes through the origin, all six sounding voices are also at the origin. But as the audience's location moves into a leaf, some voices run ahead and some voices fall behind. In effect, these six voices are in canon with one another. The time-delay between the voices constantly lengthens and shortens and thus creates a phase canon. It happens to be a canon in both pitch and timbre. Fig. 3 is a graph of time-delay vs. time for leaves one and two. An important consequence of combining the phase canons with the locational scheme is that time-delay is translated into physical distance. A given note occurs only at a particular location. As a voice runs ahead of the audience in time, it also runs ahead in location; similarly, as a voice falls behind in time, it is also behind in location. This means that from the audience's perspective, new timbres and pitches come in

Fig. 2a

Fig. 2b

Leaf #1

Leaf #2

Fig. 3

Proceedings of The 1980 International
Computer Music Conference

gradually in front while old timbres and pitches fade away behind--
ahead being the future, behind the past.

Let us put all of these formal elements together and describe how they contribute to the whole. As the piece begins, the audience is at the origin and moves out into the first leaf. As it does so, the six voices slip gradually out of phase with one another. These first notes move past the audience quickly and only gradually do notes begin to move past more slowly. As the audience moves toward the tip of the leaf, the rate of pitch and timbral change accelerates until it reaches its peak at the tip. As the audience moves back toward the origin, there are fewer notes and the voices return to synchronization. As the audience passes through the origin and into the second leaf, there is a sudden discontinuity in pitch and timbre. The second leaf begins to unfold like the first (the rate of sound movement, pitch-timbre change and spreading of the voices is the same) but, of course, new pitches and timbres are being introduced and the phase canon has a different set of time-delays. As the second leaf moves toward its conclusion, the timbral properties become similar to those heard at the beginning of the first leaf. As the audience moves again through the origin, there is a discontinuity in pitch and timbre, and the same relationships among the different elements begin to repeat their patterns. The patterns repeat for each leaf through to the fifth leaf. The form articulated by this scheme is in five parts. Probably the most recognizable repetition within the patterns is the transition from leaf to leaf. In this respect, the form resembles the construction of

the upper voices in many isorhythmic motets.

The Orchestra, Timbre and Pitch

Almost everything else important to this piece including the pitch content derives from the particular design of the single instrument that generates the sounds. The technique used for the timbral generation is a form of frequency modulation. In this case, three oscillators are connected in series, each frequency modulating the next. Although many composers have probably used this scheme for producing complex timbre in an analog studio, to my knowledge it has not been used in digital synthesis as another extension of the frequency modulation techniques first discussed by John Chowning [1]. The reason for the omission may be that the signal produced by the first two oscillators will often contain a dc component. When this dc component is used in modulating the third oscillator, it causes an erratic shift of the carrier frequency. In order to overcome the carrier shift, in front of the third oscillator I inserted a notch filter which removes the dc component and leaves the rest of the control signal untouched (Fig. 4).

This particular FM scheme has many attractive features and is very powerful in producing timbres with realistic qualities. It is particularly good with string timbres, but every type of timbre seems to benefit by having some amount of modulation on the final control signal. This scheme is, of course, capable of producing a wide range of timbres including timbres with dual fundamentals, but within this composition I limited myself to harmonic timbres (and hope to explore

Fig. 4

other possibilities in future compositions).

One of the particular advantages of this FM scheme is that it provides yet another dimension to c:m ratios; they become c:m1:m2 ratios. One can imagine that an organization of these ratios in terms similar to that done by Barry Truax [2] for c:m ratios must be substantially more complex because of the way in which the three oscillators interact. Many more subtle shades of timbre are possible with the three oscillator scheme than with two; consider that for every c:m ratio there are numerous c:m1:m2 ratios which can either reinforce or fill in the c:m ratio's spectrum. There are some additional advantages to this scheme when one considers its dynamic properties. With two modulation indices to manipulate, there are many subtle differences in types of attacks and slow spectral unfolding that are possible. Even better, manipulation of the modulation index for the two control signals provides a simple way to produce a kind of spectral fluxuation resembling that of acoustic instruments. Of course, spectral fluxuation and synchronicity constitute the dimension of timbre described by John Grey [3] which most closely correlates to the traditional instrument families. It is an extremely important factor in determining timbre and one that is not easy to simulate.

The actual instrument that is used to produce the timbres in Five-Leaf Rose is naturally more complex than that shown in Fig. 4. In addition to the parameters involved directly with the double modulation scheme, there are parameters to control random frequency and amplitude fluxuation, vibrato, detuning of harmonics, low-pass filtering, etc.--a total of fifteen individual parameters that determine timbre.

As discussed before, all of these parameters are linked to the angle θ in the geometric model. Each parameter is controlled by a function made up of the addition of two sinusoids like that shown in Fig. 5a. As the angle θ moves from zero to 360° , the values assigned to the particular acoustic parameter vary in a regular pattern. But, since the angle θ itself unfolds backwards through each leaf, the values of the acoustic parameter are also reversed within each leaf as shown in Fig. 5b. This is the specific way in which the timbral discontinuity at the beginning of each new leaf is achieved.

The functions for all of the acoustic parameters are similar in form. Examples of four functions are shown in Fig. 6. None of the functions are in synchronization with any other, so they all present a slightly different unfolding of the timbral parameter they control. The total effect of all these parameters changing in ensemble is that the timbre continuously modulates as θ changes. The combination of functions used in Five-Leaf Rose specify a particular sequence of timbres rather than all possible timbres. The sequence was arrived at through experimentation.

One of the important determinants of timbre is still the c:m1:m2 ratio. Because the range of possible ratios is nearly limitless, I adopted a scheme which reduces the choice of ratios to a limited number. Within this scheme the frequency of the middle oscillator is fixed and a sequence of ratios is created in which m1 runs from one to twelve. The c:m1 part of such a sequence would be 1:1, 1:2, 1:3, ...1:12 or 1:1, 1:2, 2:3 ...11:12 etc. The fundamentals created by any such sequence

Fig. 5a

Fig. 5b

Fig. 6

form an inversion of the harmonic series; Fig. 7 shows the series starting on c^4 . One of the compositionally useful properties of this scheme is that the ratios automatically map into pitch, that is, each unique ratio has a unique pitch. High pitches have simple ratios and low pitches have complex ratios. Another useful property is that the intervals produced in this way are the pure untempered intervals. Table 1 shows the five pitch-classes included in this series and their tuning in semi-tones and cents.

In order to expand the menu of pitches, each of the original five pitch-classes was used as the basis for another series. The pitch-classes included in each of these series and their tuning in semi-tones and cents are shown in Table 2. This expanded menu of pitches suggests a hierarchy in which certain pitches occur more often than others and it also provides a rich pallet of untempered intervals. Each leaf of the composition is based on a scale formed by combining two series; one series is always held over in common between two adjacent leaves. Table 3 shows the series combined for each leaf and indicates the series held in common. Fig. 8 shows the collections of notes formed for leaves one and five.

In spite of the fact that the scales change with each section, the melodic contour is controlled by a smoothly varying function like those controlling the acoustic parameters. After all, because of the direct relationship of pitch to the $c:m_1:m_2$ ratios, pitch and timbre are inseparable. The function for the melodic contour is shown in Fig. 9a. This function is interpreted in a freer way than any of the

Fig. 7

pitch-class	semitones . cents
c	0.00
f	4.98
a $\flat$	8.14
d $\sharp$	2.31
b $\flat$	9.96

Table 1

Proceedings of The 1980 International
Computer Music Conference

C		F		Ab	
c	0.00	f	4.98	ab	8.14
f	4.98	bb	9.96	db	1.12
ab	8.14	db	1.12	fb	4.28
dt	2.31	gt	7.29	bb	10.45
bb	9.96	eb	2.94	gb	6.10

Dt		Bb	
dt	2.31	bb	9.96
gt	7.29	eb	2.94
bb	10.45	gb	6.10
et	4.62	ot	0.27
ot	0.27	ab	7.92

Table 2

Leaf #1	C	&	D ⁺
Leaf #2	C	&	A ^b
Leaf #3	D ⁺	&	A ^b
Leaf #4	D ⁺	&	F
Leaf #5	B	&	F

Table 3

Leaf #1

C & D⁺

Leaf #5

B^b & F

Fig. 8

Fig. 9a

Fig. 9b

other functions. First, the function is taken as a general determinant of range and not of specific pitches. Second, there are numerous reversals of segments of the function within the individual leaves. These reversals are small reflections of the large-scale reversals that occur between leaves and present a melodic contour with many clearly articulated shapes. A graph of melodic contour vs. time is shown in Fig. 9b.

Having discussed the formation of the scales and outlining of melodic contour, we arrive at what for most listeners is the most immediate aspect of the composition: melody. Even though melody is logically the last aspect of composition to be discussed and even though it is the most freely determined aspect, it is an important part of the design of the whole. The freedom that distinguishes melody from all other aspects contrasts with the organization of all other parameters. That freedom is tempered by the need to make the phase canons clear, to create harmonic relationships, and to exploit the immediate timbral qualities, but it is largely composed in an intuitive fashion. Sometimes it supports the patterns of the other parameters; sometimes it counterpoints them. Fig. 10 shows the melodic lines for leaves one and five. The melody is composed with the purpose of giving each leaf a distinct character; that character is related to the scale with its tuning, and the time delays used with the phase canon.

Conclusion

The compositional perspective that I have demonstrated here should be clear. In the composition of computer-generated music it is not only possible but desirable to consider the way in which even the

1992 12

...and the *Journal of the American Medical Association* (JAMA) published a letter from the American Medical Association (AMA) to the JAMA editor, dated June 19, 1968, in which the AMA expressed its opposition to the proposed legislation.

Fig. 10

Proceedings of The 1980 International Computer Music Conference

smallest acoustic detail relates to the whole. In a sense that goes beyond other compositional media, it is possible to integrate large-scale and small-scale design and to present a single, unifying scheme for an entire work. The geometric model I have used for Five-Leaf Rose is such a scheme, but only one of many possible approaches. One of its advantages is that it is conceptually so simple, and yet can produce complex musical relationships.

References

1. John Chowning, "The Synthesis of Complex Audio Spectra by Means of Frequency Modulation," Journal of the Audio Engineering Society, Vol. 21, No. 7, 1973, pp. 526-534.
2. Barry Truax, "Organizational Techniques for C:M Ratios in Frequency Modulation," Computer Music Journal, Vol. 1, No. 4, 1977, pp. 39-45.
3. John M. Grey, An Exploration of Musical Timbre, Ph.D. Dissertation, Stanford University, 1975. Department of Music, Technical Report No. STAN-M-2.