CHAOTIC NON-LINEAR SYSTEMS AND DIGITAL SYNTHESIS: AN
EXPLORATORY STUDY

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ABSTRACT

The study of classical dynamic systems has recently undergone a major reformulation in recognizing that even when such systems are deterministic they can exhibit complex and chaotic behaviour. Such behaviour, which stems from a basic nonlinearity in the system, has been observed in a wide variety of physical and biological phenomena, but in acoustical systems one of the few applications has been to woodwind multiphonics. This paper presents examples of the application of chaotic systems to digital sound synthesis for the purpose of timbral design, as distinct from mappings onto macro-level musical parameters which has characterized most work in the past. Both one-dimensional systems, such as the logistic map, and two-dimensional ones, are presented and applied to the control of real-time granular synthesis.

1. INTRODUCTION

Over the past 20 years, the study of classical dynamic systems has undergone a major reformulation, after about three centuries of stability (Moon, 1987; Schuster, 1988). The changes have centred around the realization that even when such systems are deterministic (such as the classical forced pendulum) they can exhibit complex and chaotic behaviour. Such behaviour, which stems from a basic nonlinearity in the system, has been observed in a wide variety of physical and biological phenomena but can be accounted for by similar descriptions. What seems to have attracted the most public attention is the fact that such behaviour displays fractal properties and self-similarity across different scales. Various composers have attempted to find musical analogies to the famous computer graphics fractals, but most of these attempts have involved a mapping onto macro-level compositional parameters as in the works of Larry Austin (Clark, 1989), Charles Dodge (1988) and Bruno Degazio (1986).

Given that acoustical systems are prime examples of dissipative dynamical systems, it is surprising that more work has not been done to investigate fundamental relationships between chaotic behaviour and acoustical models of sound. The unsolved problems in the field of acoustics seem as ripe for such basic re-examination as those in other fields (e.g. turbulence) which have been completely reformulated in recent years. Gleick (1987) has suggested that sociological reasons, as well as the recent availability of the computer, may have contributed to the latency of the systematic investigation of chaotic behaviour in dynamical systems. For instance, he suggests that scientists have traditionally been trained to think in terms of linear systems and solvable linear differential equations as the norm, and to ignore the irregularities of any complex behaviour that cannot be explained by them. Are not the "difficult" problems of acoustic phenomena treated similarly? Here we can cite onset transients, departures from pure harmonicity, and the complex behaviour of certain types of environmental sounds as examples whose full explanation has eluded researchers.

However, recent work in woodwind multiphonics by Gibb (1986) and Maganza et al. (1986) has shown that these sounds are the result of chaotic behaviour in the forced oscillation of the instrument when subjected to a large amount of pressure. A useful technique which they have used to document this behaviour is the so-called 'pseudo phase space' representation of the signal where the x and y axes show the signal at time t and t+T respectively. Whereas periodic
oscillations appear as simple closed figures in such a representation, chaotic behaviour shows, for instance, a characteristic period doubling and rippling. Given that the amount of delay T in the delayed signal is usually only a few samples, this representation is easy to obtain in real-time digital synthesis with a stereo converter. The direct and delayed signals can be fed to the x and y axes of an oscilloscope.

The work in monophony suggests that other examples of non-linearity in acoustics should also be examined for chaotic behaviour. Generally, non-linearity has been regarded as an aspect of the processing of a signal such as by the ear to produce aural harmonics or difference tones) or by electroacoustic equipment (where non-linearity leads to distortion). However, in digital synthesis, a variety of nonlinear techniques, the most famous of which are frequency modulation (FM) and wave shaping, have been used to generate dynamically changing timbres (Dodge & Jerse, 1983). The time behaviour of such systems, though, is often chaotic in appearance, particularly at medium to high modulation levels. It may not be coincidental that on route to chaotic behaviour in physical systems, the mode-locking frequencies that occur as a ratio between the system’s oscillations and those of the forcing function correspond to the Fourier series, which has been shown (T table, 1977) to characterize all unique carrier to modulation ratios in FM.

II. NONLINEAR MAPS

The simplest form in which to express non-linear systems that exhibit chaotic behaviour is the difference equation in one dimension, or a set of them in more than one. The simplest and most commonly referred to single-dimension equation is the ’logistic’ or ’quadratic’ map:

\[ x(n+1) = r \cdot x(n) \cdot (1 - x(n)) \]

where the factor r ranges from 0 to 4. The non-linear factor in the equation is the \( x^2(n) \) part of the function which is subtracted from the linear term. The term ‘logistic’ is used because the equation has been used (since P. Verhulst in 1845) to model population cycles in a closed area where r depends on fertility, food supply, etc. From the acoustical point of view, the most significant aspect of the use of these equations is that they are now treated dynamically according their time behaviour, that is, according to successive iterations of the equation. The advent of the computer has made it relatively easy to track the equation’s behaviour over thousands of iterations. The results can exhibit any of the following characteristics, e.g. the movement can:

a) die away to zero
b) stabilize around a fixed value
c) oscillate in a periodic cycle
d) exhibit chaotic behaviour

The logistic map exhibits all of these behaviours according to the value of r. Below r=1, the result converges on zero; from r=1 and r<3, it converges rapidly on a fixed non-zero value. Around r=3, the values bifurcate into period 2, 4, and 8 cycles, and beyond r=3.57, chaotic behaviour occurs.

Jeff Pressing (1988) has used this function to generate musical gestures by mapping the values onto the pitch of successive notes. Washburn (1987) has attempted to extend this work to the timbral domain using MIDI-controlled synthesizers and a Synclavier II. However, a more inherently micro-level implementation seems desirable if flexible timbre generation is to be performed. Also, from a more philosophical or aesthetic point of view, it is not clear that an arbitrary mapping of a non-linear function is inherently more musical than, for instance, a random or stochastic function. The musicality may reside in the musical knowledge of the
mapper more than in the source function. The audience, if suitably primed with program notes, may be convinced there is more value or interest in the result because of the technique used, but the half-life of such interest seems to be short. Synthesis techniques of more sustained interest reflect a particular model of how sound is thought to behave and in a sense are a test of the validity of that model. If the model of 'chaotic determinism' is to have lasting value in sound synthesis, it must reflect certain truths about actual sound, even within its role as simply a model. The truth here would seem to be that complex behaviour can result from a simple set of acoustical factors.

It is relatively easy to apply the one-dimensional logistic map to the raw of phase change of a simple oscillator, that is, to control its instantaneous frequency. The result depends on the time scale at which the equation is re-iterated. When the iteration is applied at intervals less than the period of the waveform, values of r between 1 and 3 produce a transient behaviour as the oscillation settles down to a fixed frequency. Beyond r=3, successive period doubling results in a frequency modulated signal, and in the chaotic range, the limitation to single-precision calculation results in complex periodicities rather than actual non-repeating chaos.

However, the fixed waveform oscillator itself is a rather poor model of acoustic reality, and therefore, a more interesting application of the logistic map is found with real-time granular synthesis (Truxx 1988). The frequencies of successive grains can be calculated according to the equation with the factor r as a user-controlled variable. Here, the dynamic aspect of the grains (which embody both frequency and time variation) combines with the dynamic interest of the non-linear map to produce sonically interesting results. Since grain durations are typically in the range of 10-50 ms, the rate of iteration of a chaotic function lies between the micro-level of phase change and the macro-level of conventional compositional parameters. The result is perceived as a chaotic texture, analogous to turbulence or other non-linear phenomena.

The best known two-dimensional maps (involving pairs of equations for x and y) are the Hénon map and the so-called 'chaotic gingerbread man' (Devaney, 1988), as well as the Julia set in the complex domain. With appropriate scaling for differences in frequency and time axes, these maps can also be imbedded within the granular synthesis model to integrate both of its basic variables. Of particular interest would be functions where frequency and duration are inversely proportional as suggested by Gabor (1947) to be appropriate for a 'quantum' approach to acoustics. However, in several of these two-dimensional systems, the y value is merely the previous iteration of the x value, possibly scaled. Therefore the equation can be reduced to a single dimension with a one sample time delay, as in a first-order filter equation. For instance, the 'chaotic gingerbread man' can be expressed as:

\[ x(n+1) = 1 - x(n-1) + 1/(n) \]

This version of the equation allows it to be used in a granular synthesis implementation to calculate the frequency of successive grains. Further variations of the equation can be obtained by scaling the non-linear element (the absolute value of x(n)) by a factor r, similar to the logistic map. In each case, another characteristic of chaotic behaviour - sensitivity to initial conditions - adds additional variety since the resulting sequence depends on the two initial values which can be randomly generated. Some values produce a periodic oscillation while others produce local periodicities with chaotic excursions.

III. CONCLUSION:

Chaotic non-linear systems are currently attracting a great deal of attention from the non-scientific community, including artists who seek parallels in their own field. For the computer composer, the question remains as to whether there are fundamentally non-linear phenomena.
in acoustics which can impact on digital synthesis (it is highly suggestive that there are), we
whether all attempts to model chaos are artificial constructs. By analogy, we cannot say that
the music of J.S. Bach is great because it is the aural equivalent of Cartesian geometry, but we
can hardly deny that it arises from the same Zeitgeist or whatever one chooses to call the nexus
of intellectual, cultural and aesthetic currents that influence an artist. Similarly, new musical
models will undoubtedly arise from the intellectual milieu that includes fractal geometry and
chaotic non-linear systems. But will such music come from arbitrary mappings of fractal
geometry onto conventional musical parameters which seem plagued by the internal
contradiction of a marriage of unrelated partners? Granular synthesis, on the other hand, by
refocusing attention on micro-level audio domain behaviour, is a suggestive alternative.

References


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