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DIGITAL SYNTHESIS OF COMPLEX SPECTRA BY MEANS OF MULTIPLICATION OF NON LINEAR DISTORTED SINE WAVES

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INTRODUCTION.

Synthesizing a sound by digital methods, for example using the Music V program (1) or special digital circuits, implies the definition of the mathematical procedure to be followed for constructing the sound, and for specifying its parameters. For example, additive synthesis consists of adding sine waves, the amplitudes of which are functions of time. Subtractive synthesis corresponds to the digital filtering of a complex waveform, wherein one changes the filter characteristics with time. However, with these two methods one must be painstaking to obtain sounds with rich musical quality and various playing possibilities. Recent research has focused on global methods of synthesis, which can be called modulation methods : a sound is defined by the evolution in time of one amplitude, one or more frequencies determining a line spectrum and one or more indexes of timbre determining the amplitude of these partials. This class of methods includes the FM synthesis technique described by Chowning (2) where the time variables are the amplitude, the carrier and modulation frequencies and the modulation index(es), and the discrete summation formula method described by Moorer (3,4) where the variables are the amplitude, the starting frequency, the frequency difference between partials and one index controlling the ratio of the successive partials. The power of such methods depends upon their ability to generate a palatte of related but distinct sounds. Similarly, in the syntax of the Music V program, there are two levels : the instrument (INS definition) which can define a class of sounds, and the notes parameters (NOT), which can define one specific sound inside this class. In this sense, a global method is powerful if one can define differentiated instruments whith enough possible articulations in notes. Non linear distortion of sine waves (also called wave shaping) is a global method, which can be introduced as follows : the distortion of a sine wave by a "bad" amplifier produces a number of harmonics, the amplitude of which depend upon the amplitude of the input and the non linearity of the amplifier. This technique has been already used in computer music (Risset (5) generated clarinet-like sounds by using a simulation of a saturated amplifier). A first improvement is to use a transfer function in terms of a limited degree polynomial, so that the input is band limited, eliminating foldover (1). This method uses the fact that it is possible to calculate a transfer function of this form transforming a given input value into a given output spectrum. Conversely it is possible to calculate the output spectrum from every transfer function and input value (Shaefer (6), Suen (7)). For a given transfer function, the variation of the input amplitude (which we will now call the index fonction) leads to a variation of the spectral balance of the output. The knowledge of a transfer function allows us to determine an instrument, i.e. a class of sounds. Notes of different qualities are produced by choosing different index functions.

The first part of this paper deals with the standard use of non linear distortion. Although self sufficient, this method becomes more interesting when we add amplitude modulation (multiplication of the output by a sine wave) thereby shifting the spectrum and folding it through zero (as in FM). As we shall explain, this allows us to produce spectra with formant structures, missing harmonics and inharmonic sounds.

By multiplying more signals (which can be distorted or not), one can obtain more complex sounds. For example, an equivalent to the double frequency modulation (8) is presented, whereby little additional computation yields complex sounds. Some simple examples are explicitly given and some indications for producing more elaborate sounds are mentioned.

THE THEORY.

The starting point of calculation has been clearly exposed by Shaefer (6) for a fixed output spectrum and by Suen (7) for an evolving input. Here we give a presentation of their results in matrix form, which may be easier to understand and which is certainly easier to program.

Given a non linear amplifier, the transfer function of which is F(I) and an input I(T), the output is F(I(T)). If now the input is a cosine wave of amplitude X and the transfer function a nth order polynomial, then we can write :

$$(1)\quad \begin{cases} I(t) = X\cos(\omega t) \\ F(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3 + \cdots + d_n x^n \\ O(t) = d_0 + d_1 X\cos\omega t + \cdots + d_n X^n \cos^n \omega t \end{cases}$$

Developping $\cos^k WT$ in terms of cos KWT yields the following relations where H_i is the amplitude of the i^{th} harmonic, we obtain :

$$(2)\quad O(t) = 1/2\, H_0 + H_1 \cos\omega t + \cdots + H_n \cos n\omega t$$

with :

$$(3)\quad \begin{pmatrix} H_0 \\ H_1 \\ H_2 \\ \vdots \\ H_n \end{pmatrix} = 2 \cdot A \cdot \begin{pmatrix} D_0 \cdot (X/2)^0 \\ D_1 \cdot (X/2)^1 \\ D_2 \cdot (X/2)^2 \\ \\ D_n \cdot (X/2)^n \end{pmatrix}$$

A is an (N + 1, N + 1) matrix, which is abstracted from a general 2 dimensional array. Figure 1 gives the generative relations and presents the first ten lines and columns of this matrix. A practical example is also given which shows how to calculate the output spectrum partials for a given distortion and one index value.

A transfer function of degree N gives a spectrum composed of N harmonics (which is very important in digital synthesis, because this can avoid foldover, provided the sampling frequency is greater than twice the nth harmonic frequency value). There is complete independence between the odd harmonics (depending upon the values of the odd order coefficients of the polynomial) and the even ones. This can be seen in the A matrix, for A_{ij} equals 0 if I and J does not have the same parity. This feature in particular permits an easy control of the balande between odd and even harmonics in a spectrum.

Conversely, the expression of $\cos(K\omega t)$ in terms of $\cos^K(\omega t)$ gives the relations between an output spectrum for a given value of the index X and the resulting d_i coefficients.

$$(4)\qquad \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_n \end{pmatrix} = \frac{1}{2} \begin{pmatrix} (2/X)^0 \\ (2/X)^1 \\ (2/X)^2 \\ \vdots \\ (2/X)^n \end{pmatrix} \cdot B \cdot \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_n \end{pmatrix}$$

B is an (N+1, N+1) matix, the generative procedure and a(10,10) subset of which is given in figure 2. A practical example indicates how to calculate the polynomial coefficients to obtain a given output spectrum for one value of the index.

THE STANDARD USE OF NON LINEAR DISTORTION.

The preceeding section has shown how to calculate a transfer function F(I). This allows us to produce for a given input of amplitude X an output spectrum with given values of the first N harmonics, using a polynomial of nth order. Our concern is now the evolution of the output spectrum in terms of the evolution of the X input.

With the same transfer function we can calculate every spectrum corresponding to other values of the input amplitude X (which is in fact a timbre index). Each sound produced this way has no more than N harmonics, and the evolution of the spectrum with X presents no discontinuity, though it can present individual large variations of the harmonics, including zeroing and changing of sign. As indicated earlier, the evolution of odd and even harmonics are absolutely unrelated (as a limit example, setting to zero the even order coefficients terms of the polynomial insures every spectrum to comprise only odd harmonics). For any transfer function, we can draw the evolution of a spectrum as a function of the input amplitude. A possible representation is presented in figure 3 were the evolution of the spectrum is computed for different values of the index.

THE STANDARD USE OF NON LINEAR DISTORTION IN DIGITAL SYNTHESIS.

To perform non linear distorsion, a sound synthesis program or a digital device requires a polynomial function generator in order to get a table corresponding to the transfer function, and the simulation of a distorting amplifier through a table look-up device.

THE POLYNOMIAL FUNCTION GENERATOR.

A function generator has to compute a table of values corresponding to the transfer function, the input sequence being the coefficients D0, D1, D2,...,DN of the polynomial. Due to the fact that the function is digitally stored in a limited area and its absolute value must be bound, the generated function must be centered and normalized ; hence we have choosen to store the function in a (2M + 1) area and to compute :

$$F(j) = \left(\sum_{i=0}^{m} d_i \left(\frac{j}{m}\right)^i \right) / T$$

T is a normalisation value such that the maximum value of this function is 1. It must be clear that with this definition we can relate X and J by :

$$J = m \cdot X \qquad \text{and} \qquad X = \frac{J}{m}$$

and so the excursion of the index of timbre X is limited to (-1,1) as stated in figure 4. This has proved not to be a drastic limitation, because one can transform the original mathematical polynomial and input by the change of variables $\begin{cases} X' = X/XMAX \\ D_i' = D_i \, X_{MAX}^{\,i} \end{cases}$

THE TABLE LOOK-UP UNIT GENERATOR.

We now need a table look-up device (or program). It must calculate an output from two inputs corresponding to : OUT = I1·F(I2). If it is calculated without interpolation (by truncation) the table must be choosen large enough to avoid a kind of quantification noise. An other feature is that the origin of the scanning is not the first location but the M + 1 one, so the input can be negative.

AN INTRUMENT FOR STANDARD USE OF NON LINEAR DISTORTION.

We give here more specific details in the framework of a Music V implementation. A function generator has been written, which calculates a polynomial function the values of which are stored in a 512 locations area. Since it is not an odd value, they are effectively stored in 511 locations (figure 4), so the preceeding value of M is 255. A new unit generator has been written, which is a table look-up with interpolation. The standard Music V oscillator could have been used in a degenerated way, with a null increment, however this is cumbersome. An instrument following Mathews definitions can now be defined in figure 5.

The index functions (F1) is intended primarly to describe the variation of timbre with time. However the loudness is also a function of the index and care must be taken in the appreciation of the amplitude value (P5) that will often have to be variable and not fixed as in figure 5. Some automatic correction could be calculated by the means of a normalization function as discussed by Moorer (3). However it requires storage for a new function and computation time, it complicates the instrument and it is only approximate. Moreover this variation of loudness often sounds natural and the simplest way to use this instrument is to use if necessary the P5 input as a post compensation of loudness.

The transfer function (F3) generates the distortion and consequently it determines the harmonic content of the output as a function of the input. In general the timbre becomes richer as the index increases, but this is highly dependent on the choise of this transfer function. However some general remarks can be made : in the standard use of the non linear distorsion the d_o term (the constant of the polynomial) should be set to zero to avoid a click at the beginning and end of a note especially with softer (small index value) notes ; the odd and even harmonics are independent and the maximum number of harmonics is equal to the degree of the polynomial (but the higher harmonics may be insignificant). When the order is small one can use the direct equalities (3) to predict this evolution. With larger values one needs the aid of a computer program displaying the evolution of the spectrum as a function of the index value.

THE CHOICE OF A TRANSFERT FUNCTION.

The characteristics of the non linear distortion are entirely determined by the coefficients of the polynomial. But there are many ways to calculate these coefficients.

1.- Evaluation from a steady-state spectrum.

Relation 4 allows us to calculate the D coefficients (the polynomial) starting from the H ones (the spectrum) given an index value X0. We have seen that the distortion of a cosine wave gives only cosine components, the amplitude of which are real but can be positive or negative. So different distortions, hence different timbre variations, can be used which verify the initial conditions, only by changing the signs of some components. An abruptly limited spectrum or a rich spectrum (more than 20 harmonics) often gives rise to irregularities in the evolution of the sound with the index, which is akin to the Gibbs phenomenon.

2.- Evaluation from a continuous transfer function.

In this case, the general form of the transfer function is known, and the problem is to get a polynomial approximation of it, in order to obtain a band limited spectrum and avoid foldover. A limited development of the function approximates it very well around the origin (small values of the index) but usually not in other areas. One can also take the complete transfer function, calculate the spectrum for a specific index value and then use the first procedure with the first N harmonics. This approximates well the spectrum for that value of the index, but abrupt bandwidth limitation causes the ripple effect previously described and it may prove useful to attenuate little by little the last partials values to get smoother evolutions with almost identical spectra (this is in effect a kind of windowing). Other classical algorithms can be used, such as the least mean square approximation.

3.- Direct evaluation.

An experimental and/or intuitive choice of the coefficients of the polynomial is also a good strategy : there is a strong but subtle connection between the regularity of variation of the coefficients (and their sign) and the homogenity of the resulting timbre. Choosing to affect the same sign to all odd order coefficients, and doing so for even order coefficients (though this sign may be different of the other one) produces a very steep transfer function, which may produce too brassy sounds. A good practise can be to alternate the sign of successive odd order coefficients (say A1, A5, A9 positive, and A3, A7, A11 negative) and do the same for even order coefficients.

SOME EXAMPLES.

Examples 1 to 4 use the previously described Music V instrument, except that "amplitude" input (for correction of loudness) comes from an oscillator scanning once the compensation function F4.

Example 1. Brilliant sounds.

Many non linear distortion create impulse like waveforms (figure 6). In this case, the timbre becomes richer as the index increases.

Example 2. Clarinet-like sounds.

We have determined a distortion producing a sound with only odd harmonics, becoming richer as the index increases and able to product 25 harmonics without ripple phenomenon. (Figure 7).

Example 3. Spectra merging into one harmonic.

A Chebyshev function of nth order produces a distortion where the spectrum is restricted to harmonic number N when the index is one. Let us assume that the index evolves as shown in figure 8. If N is odd, only odd harmonics are produced in the intermediate state ; if is even, the sound is an octave higher. With respect to this higher fundamental, harmonic number N/2 is produced at the end, the intermediate state comprising both odd and even harmocis (figure 8).

Example 4. Sounds merging into a group of harmonics.

A formant-like spectrum can be achieved for example with the function described in figure 9, which has been calculated to give a formant spectrum for an X value of .8. The sound described in this figure evolves from a simple sine wave to this formant-like spectrum. The sound of figure 10 yields a percussive spectrum with a complex resonance by keeping the same distortion and by choosing a different index evolution.

SHIFTING AND FOLDING THE SPECTRUM.

Through amplitude modulation, the spectrum can be shifted to any desired center, thus folding around zero some components of the sound. In the Music V format, this corresponds to the multiplication of the output of our proceeding instrument by a sine wave at frequency f_{am} ; it can be represented as in figure 11. A Music V instrument may be the one of figure 12, in which the additional parameters are the amplitude modulation frequency and the initial phase (P 8).

HOW AMPLITUDE MODULATION WORKS.

What happens to the components of the original sound because of amplitude modulation can be explained in the following way : the multiplication of time functions is equivalent to the convolution of their spectr In our case, the resulting spectrum is the complex sum of the spectrum produced by the non-linear distortion shifted to right by the quantity fam and also shifted in phase, and of the same spectrum shifted to the left and reverse shifted in phase (figure 13). Taking as we did, the non-linear

distortion of a cosine wave rather than a sine wave permits us to understand more easily the importance of the phase of the modulation (P 8 in our instrument) :

- If the amplitude modulation uses a cosine wave, as :

$$\cos(wt) = \frac{1}{2}\left(e^{jwt} + e^{-jwt}\right)$$

The resulting spectrum is half the algebric sum of the two oppositely shifted spectra, because the initial distortion produces only cosine terms.

- If the amplitude modulation uses a sine wave, as :

$$\sin(wt) = \frac{1}{2}\left(e^{jwt} - e^{-jwt}\right)$$

the resulting spectrum is half the algebric difference of the two oppositely shifted spectra, and afterwards phase shifted by $\pi/2$. The phase has importance only if folded components are covering one another. A D.C. component can produce clicks and in this case, it is important to note that using a sine modulation wave eliminates this D.C. compoment, to avoid cliks at the beginning and the end of a note.

HARMONIC AND INHARMONIC SPECTRA.

We have seen that the spectrum thus shifted has components distributed around the amplitude modulation frequency f_{am}, spaced at intervals of F1 (frequency of the undistorted sine wave). The components folded through zero may be thought as originating of parts of the left spectrum (figure 13).

The use of amplitude modulation is very close to the FM synthesis technique. Producing an harmonic spectrum needs :

$$\frac{F1}{F_{AM}} = \frac{N1}{N2} \qquad (5)$$

(where N1 and N2 are mutually prime) or other written :

$$F1 = N_1 F_0$$
$$F_{AM} = N_2 F_0$$

where F0 is the resulting fundamental frequency.
N1 and N2 being mutually prime, there is an exact superposition (and thus

the phase becomes important) only if N1 is one or two. Figure 14 shows the aspect of the spectrum for different values of N1 and N2, without any consideration of amplitude and phase values (these depend of the nature of the distortion used).

Inharmonic spectra are produced if F1/FAM cannot be expressed as a ratio of integers, that is to say if folded components are not in simple harmonic relations with direct ones (figure 15). If F1/FAM approaches a simple fraction, some roughness or even beats can be heard, but the main impression is one of harmonicity (though no exact rational fraction can be written). If F1/FAM is represented by a non-simple fraction (the limit being dictated by perception) inharmonicity can be heard, though the spectrum is quite harmonic (but with a sparse spectrum) and even noisy sounds can be produced this way (for example by keeping N1 = 1 and taking N2 = 10 or 20 or even more).

The effect of the constant term of the polynomial is transfered to the carrier component. For a zero value of the index, if D0 is non zero, it produces a sine wave at the amplitude modulation frequency. The presence of clicks also depends on the amplitude enveloppe. If the amplitude (post-compensation) rises and falls slowly, there will be no clicks.

CHOOSING THE AMPLITUDE MODULATION.

Choosing the carrier frequency (amplitude modulation frequency fam) and the modulation frequency (input frequency F1) gives the pattern on which the non-linear distorsion spectrum will be inscribed, shifted and folded through zero. As was suggested by Maillard (9) for FM, one obtain a good image of what happens by drawing on a transparent paper a frequency axis, marking a point at the carrier frequency and distributing N equidistant points right and left of this point (N is the degree of the polynomial). These points may fall in the negative frequency domain. Then draw a spectrum, reporting the harmonic lines of the distorted cosine wave symetrically about the AM frequency (if the output expression is, as stated, 1/2 H0 + H1 cos WT + ... HN COS NWT, draw a H0 component at the carrier frequency, H1 at the immediate left and right, and so on). Now if the modulation wave is a cosine all you have to do is to fold your transparent paper at the frequency origin, thus obtaining the folded components to be added. If the modulation uses a

sine wave, these folded components must be subtracted. If there is no coincidence between the two parts of the paper, the phase is of no real acoustical importance. If there is a component at the origin, take it without modification (cosine modulation) or not at all (sine modulation). the latter case avoids the formation of possible clicks for the D.C. component is always zero. However, even with cosine modulation, clicks due to D.C. component are rarely audible for this component does not exist for small values of the index. Now if we use a modulation wave between sine and cosine, components are to be added in the complex frequency domain, one shifted and the other reversely shifted.

SOME EXAMPLES.

As for standard use, they are crude examples, that is to say they use nothing but non linear distortion and amplitude modulation. These processes must be made more complex to get musical sounds, as will be developped further.

Example 5. Sounds with resonances.

By shifting a spectrum up to a high harmonic, we can simulate the equivalent of a filter of decreasing bandwidth through the tone (figure 16).

Example 6. Percussive sounds.

An inharmonic (and somewhat noisy) percussive sound accur if the partial around which the formant-like structure lies is higher than before. This is obtain by changing the modulation frequency (figure 17).

Example 7. Plucked sounds.

If the formant structure is lower situated, the sound has the dual property of becoming thin but centered higher in frequency (figure 18) when the index becomes smaller.

Example 8. Clarinet-like sounds.

Odd harmonics only can be produced by an ordinary distortion (with odd and even terms in the polynomial) by choosing F1 = 2 FAM (figure 19) or F1 = 4 FAM.

Example 9. Bell or chimes imitations. (Figure 20).

Many inharmonics sounds can be obtained by using a non rational value for F1/FAM, for example sqrt (2) (this is a useful number because the folded components do not lie too close to zero or the initial fondamental frequency F0). With a fixed choise of carrier and initial fondamental frequency, one can choose different distortion, which makes possible to play timbres melodies.

DOUBLE MODULATION AND MISCALLANEOUS.

Chowning (8) uses a double frequency modulation, so that the frequencies of the partials are now $\pm$ FC $\pm$ FM1 $\pm$ FM2. A good description of the use of double FM for the simulation of piano and violin sounds is found in Schottstaedt (11). This way of proceeding is a real improvement over classical FM and one can ask if it is possible to find an equivalent with non linear distortion. This result can be obtained by introducing amplitude modulation by an evolving complex wave. It consists (figure 21) in multiplying the output of two standard use instruments (so partials frequencies are $\pm\alpha$F1 $\pm\beta$F2 if the frequencies of the undistorted signals are F1 and F2). To get the exact equivalent of the double FM, one has to multiply the output of two distorted waves and a pure sine wave so partials frequencies are now $\pm$ FAM $\pm\alpha$F1 $\pm\beta$F2.

The interest of such a technique is twofold :

- using FAM, F1 and F2 values such that they provide an harmonic spectrum (all multiples of a fundamental frequency) produces an extension of the harmonic content, keeping the order of the two polynomials low, hence more easily predictable. It is indeed hard to find musically good distortion polynomials with a rich spectrum, because they tend to be brassy or noisy or to have ripple in the evolution. This problem is entirely solved by a double modulation.

- using irrational ratios for FAM,F1 and F2 provides an incredible richness of inharmonic sounds, the problem being in that case to distinguish which of the three frequencies, two indexes and two distortion functions is the actual predominant parameter, and how to choose them.

SIDE ASPECTS OF THE DOUBLE MODULATION TECHNIQUE.

1.- A subset of the "double modulation instrument" described in figure 21 is obtained by making index 2 constant, that is to say the output of the distortion is a complex but fixed waveform. This is therefore equivalent

to using the "normal amplitude modulation instrument" described in figure 12, the sinusoidal modulation being changed to a more complex one. The resulting spectrum is the convolution of the distorted spectrum by the modulating one, it is obtained by replacing each component of the first spectrum by the entire second spectrum. In tha case where F1 is greater than FAM, the spectrum takes the appearance represented in figure 23.

2.- If F1 = F2, the output of the double distortion standard instrument is equivalent to the distortion of a sine wave by a distortion polynomial that is the product of the two distortion polynomial used. This is a way calculating a polynomial with a large order. More trivially it indicates that taking the square or the cube of the output of a standard use instrument extends its spectrum by a factor of two or three, but more generally there is a difference between using two polynomials and one because, in the first case, there are two different indexes that can do different things.

OTHER WAYS.

We may ask what happens to our previous shemas if one uses other waves than sinusoids as input in a distortion unit. We can predict that a waveform made up of M harmonics will be distorded to produce a sound with (M x N) harmonics, but the analytical solution is not simple, when M and N are large. A particular and interesting case is when the input is in the form I/2 (1 - cos WT) = I $\cos^2$(W/2 T), for using the distorsion polynomial D0 + D1 . X + D2 . X2 + ... is equivalent to using the distortion of $\sqrt{I}\ \cos\frac{w}{2}t$ by $d_0 + d_1 x^2 + d_2 x^4 + d_3 x^6$ so in this particular case we see it is equivalent to use this input or to change the distortion ; with that kind of input (or seamilingly, with the second distortion, the odd end even harmonics of the fundamental are no longer independent. This can be a useful feature if one gets into trouble with odd harmonics going in one way and even ones in another with an usual input (for in this case, these two parts may be heard as being two separate tones). It can even be the basis of a new system where all inputs are of that form, so the new polynomial generator has only to provide output for positive values of input, and the add generator has to be excluded from the instruments. This can be advantageous for hardware implementations.

A CONVERSATIONAL IMPLEMENTATION.

We now give some details on a conversational implementation of multiplication of distorted sine waves on a minicomputer. Music V program has been implemented by F.Nayroles at the computer and musical acoustics department of the Laboratoire de Mécanique et d'Acoustique appliquée in Marseille.

While this implementation on a small 16 bits minicomputer is useful, the turn around time is quite slow for trial and error tests. In addition, ther are four D A 14 bits multiplexed converters (using direct memory access) and a real time external clock, all entirely designed and constructed by P.Karatchenzef. The sampling frequency, the start and stop of conversion, and other parameters are software controlled. An interesting feature is the ability to cycle on a buffer ; it has been implemented as follows : the buffer (taken as 100 points) is continuously scanned and converted by hardware ; it contains the values of a distorted sine wave and it can be renewed as too fast as a new waveform can be computed (figure 22). This takes 17 MS on our minicomputer (floating point multiplication : 25 μS). A Fortran interactive program has been written (with an internal assembly langage inner loop) for performing non linear distortion. It permits the user to define sounds generated by a standard or amplitude modulated instrument. FAM/F1 must be a rational fraction. This is possible because the rate of scanning of the buffer can be programmed and is 100 times (the length of the buffer) the fundamental frequency.

Specifically, this is a conversational program to calculate a distortion, display spectra, accept amplitude compensation, index functions, values of AM and input frequencies (in terms of N1 F0 and N2 F0) and produce sounds in real time (the sound is calculated during its execution, but the parameters are prepared in advance). This gives a non hi-fi quality sound (a 17 MS calculation time gives some roughness in transients) and one must adjust the sampling filters to 50 times the fundamental frequency. However it is very efficient as a trial and error method, and the Music V values can be directly derived therefrom.

CONCLUSION.

The general principles presented here can lead to many developments in digital synthesis of music : the description and use of the instruments is fairly simple, so one can concentrate on musical aspects of the sounds. For example one can use a "timbre vibrato" instead of a frequency or amplitude vibrato by adding to the index a sinusoidal function. A little offset in amplitude modulation or input frequencies provides a slight inharmonicity (if FAM is high, the interval between harmonics is reduced or expanded) or even beats (if FAM is low, folded components can beat with direct ones). Certainly adding planned or random modulation upon the frequency or portamento glides (as D.Morrill (10) did with a FM trumpet) helps give life to a sound, and changing this from one note to another prevents monotony. For piano like sounds or bells, doubling or tripling the notes with small and different frequency differences makes the sound more natural. Much work can be done in this direction.

In comparison with the classical FM technique (2) there are some differences : the non-linear distortion is exactly band limited, whereas FM is not ; the frequency modulation with sine wave oscillations is unique, the non-linear distortion can be varied by changing the distortion polynomial (to the extent that only changing this polynomial in a Music V score gives another "orchestration" feeling) ; also the missing carrier method has been widely exploited (under the name standard use) for the non-linear distortion but not so much for FM (however it exists as sin(I sin(WT + ψ) and is simply another type of non-linear distortion). Clearly other trigonometric modulations have overcome the limitations of the classical FM and are conceptually very similar to the non-linear distortion. The most important difficulty now is to find parameters corresponding to a desired musical effect. The non-linear distortion can be helpful in achieving that goal.

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The recursion formula (where i is the line and j the column numbers) for calculating A is

a(1,1) = 1

i ≠ 1   a(i,j) = a(i-1,j-1) + a(i+1,j-1)

i = 1   a(1,j) = 2 * a(2,j-1)

The easier way to compute these relations is to set the first column and to calculate successive columns.

$$
A = \begin{bmatrix}
1 & \cdot & 2 & \cdot & 6 & \cdot & 20 & \cdot & 70 & \cdot \\
 & 1 & \cdot & 3 & \cdot & 10 & \cdot & 35 & \cdot & 126 \\
 & & 1 & \cdot & 4 & \cdot & 15 & \cdot & 56 & \cdot \\
 & & & 1 & \cdot & 5 & \cdot & 21 & \cdot & 84 \\
 & & & & 1 & \cdot & 6 & \cdot & 28 & \cdot \\
 & & & & & 1 & \cdot & 7 & \cdot & 36 \\
 & & & & & & 1 & \cdot & 8 & \cdot \\
 & & & & & & & 1 & \cdot & 9 \\
 & & & & & & & & 1 & \cdot \\
 & & & & & & & & & 1
\end{bmatrix}
$$

$P(x) = x + 2x^2 + 3x^3$

calculation of H

for index X = .3

$$
\begin{pmatrix} H_0 \\ H_1 \\ H_2 \\ H_3 \end{pmatrix} = 2 * \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 * 1 \\ 1 * .3 \\ 2 * .09 \\ 3 * .027 \end{pmatrix}
$$

Fig 1 : Computing and use of the A matrix

For the matrix B, the first line is as follows :

1   0   -2   0   +2   0   -2   0   +2   ......

the other lines can be computed according to the recursion formula :

b(i,j) = b(i-1,j-1) - b(i,j-2)

$$
B = \begin{bmatrix}
1 & \cdot & -2 & \cdot & 2 & \cdot & -2 & \cdot & 2 & \cdot \\
 & 1 & \cdot & -3 & \cdot & 5 & \cdot & -7 & \cdot & 9 \\
 & & 1 & \cdot & -4 & \cdot & 9 & \cdot & -16 & \cdot \\
 & & & 1 & \cdot & -5 & \cdot & 14 & \cdot & -30 \\
 & & & & 1 & \cdot & -6 & \cdot & 20 & \\
 & & & & & 1 & \cdot & -7 & \cdot & 27 \\
 & & & & & & 1 & \cdot & -8 & \\
 & & & & & & & 1 & \cdot & -9 \\
 & & & & & & & & 1 & \cdot \\
 & & & & & & & & & 1
\end{bmatrix}
$$

Output spectrum

$$
\begin{pmatrix} H_0 = 0 \\ H_1 = .11 \\ H_2 = .12 \\ H_3 = .13 \end{pmatrix} \text{For } X = .5 \quad \Rightarrow \quad \begin{pmatrix} D_0 \\ D_1 \\ D_2 \\ D_3 \end{pmatrix} = \frac{1}{2} \underbrace{\begin{pmatrix} 1 \\ 4 \\ 16 \\ 64 \end{pmatrix}}_{\left(\frac{2}{X}\right)^i} \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ .11 \\ .12 \\ .13 \end{pmatrix}
$$

Fig 2 : Computing and use of the B matrix

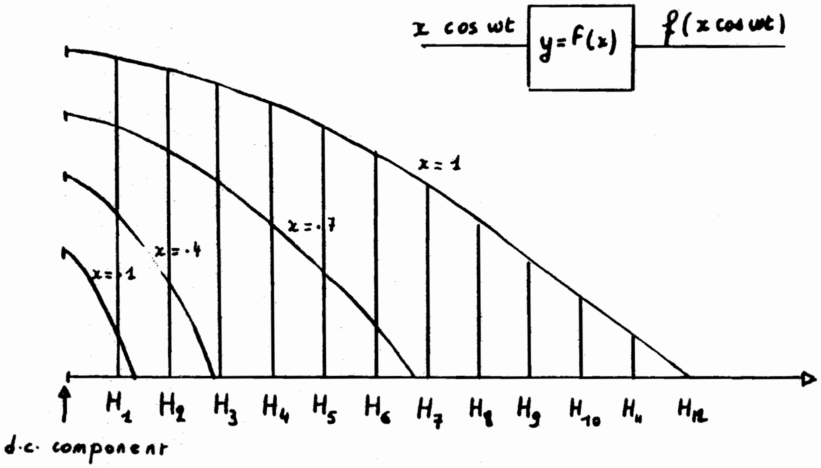


Fig 3 : evolution of the output spectrum as a function of the index

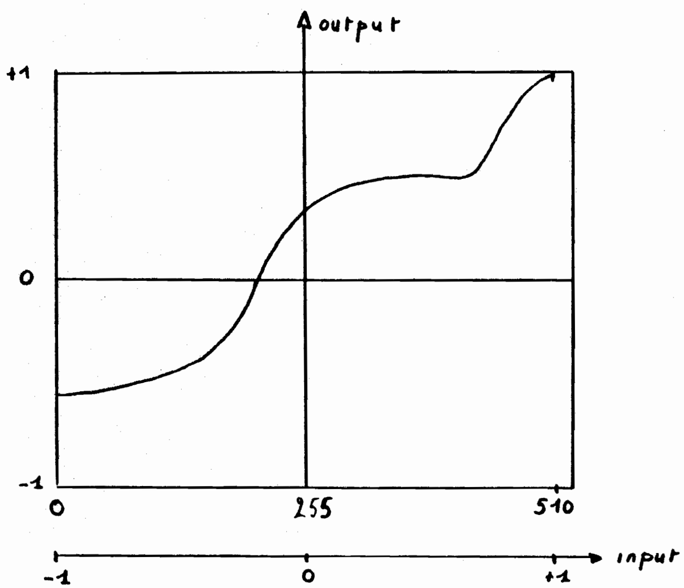
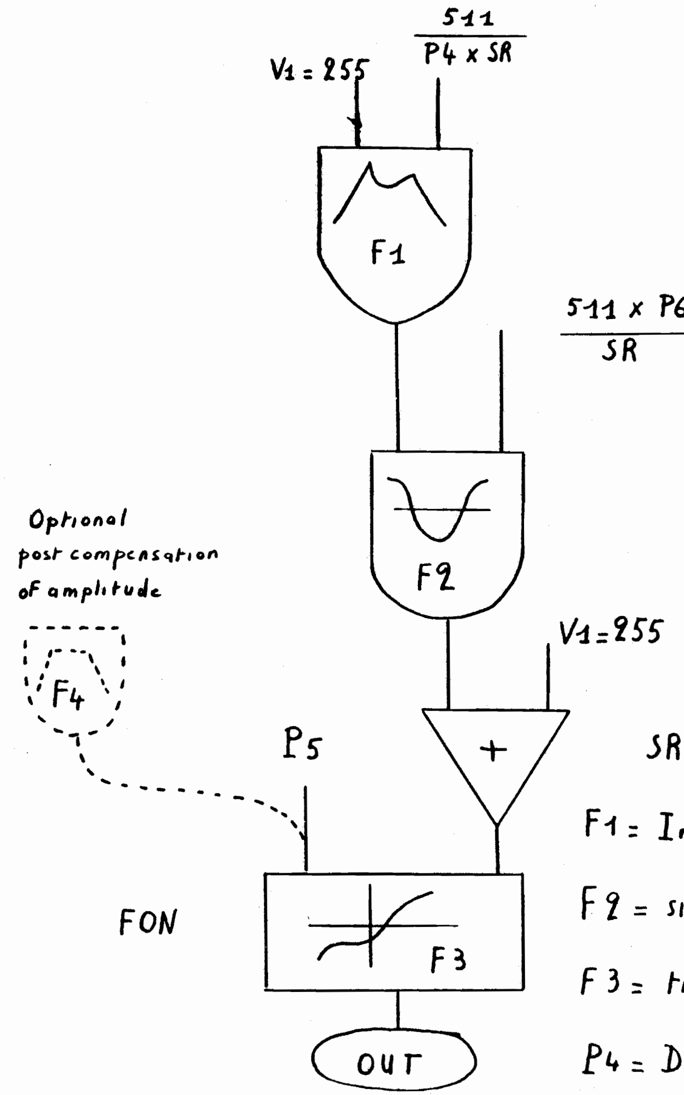


Fig 4 : The table containing the transfer function



SR = Sampling rate

F1 = Index variation

F2 = sine or cosine

F3 = transfer function

P4 = Duration (sec.)

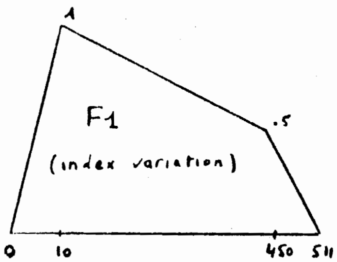
P5 = Amplitude

P6 = Frequency (Hz)

Fig 5 : a MUSIC V instrument

$400\,Hz < P6 < 1000\,Hz$

$P4 = 1s$



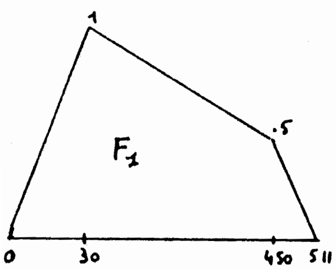
$$P(x) = x + .9x^2 + .8x^3 + .72x^4 + .63x^5 + .55x^6 + .50x^7 + .46x^8 + .43x^9 + .40x^{10} + .38x^{4} + .36x^{12} + .34x^{13} + .32x^{14} + .30x^{15} + .28x^{16} + .26x^{17} + .24x^{18} + .22x^{19} + .20x^{20} + .18x^{21} + .16x^{22} + .14x^{23} + .12x^{24} + .08x^{25} + .06x^{26} + .04x^{27} + .02x^{28}$$

P5 Fixed

Fig 6: Brilliant sounds

$125\,Hz < P6 < 500\,Hz$

$P_4 = 1s$



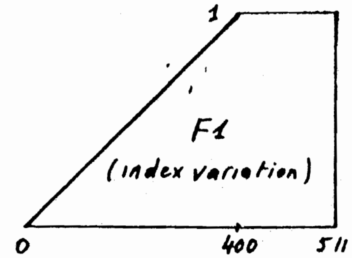
$$P(x) = 5.141x - 33.246x^3 + 176.02x^5 - 668.494x^7 + 1857.797x^9 - 3848.826x^{11} + 5996.932x^{13} - 7017.083x^{15} + 6085.882x^{17} - 3801.572x^{19} + 1619.364x^{21} - 421.588x^{23} + 50.66x^{25}$$

P5 Fixed

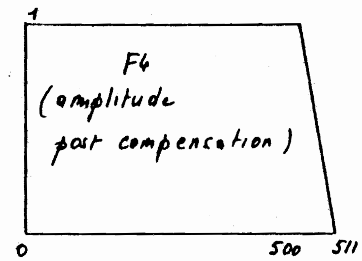
Fig 7: Clarinet-like sounds

$P4 = 10s$

$P6 = 261\,Hz$ (C4)



$$P_9(x) = 9x - 120x^3 + 432x^5 - 576x^7 + 256x^9$$



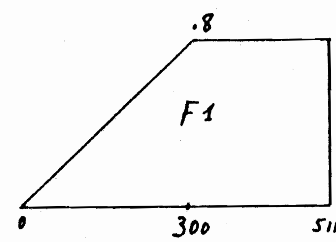
$$P_{10}(x) = 0 + 50x^2 - 400x^4 + 1120x^6 - 1280x^8 + 512x^{10}$$

D_0 set to zero to avoid a click

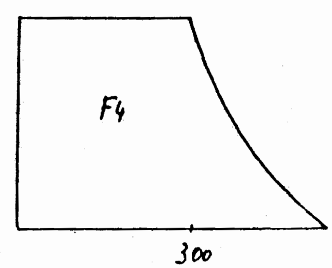
Fig 8: use of CHEBYSHEV-like polynomials

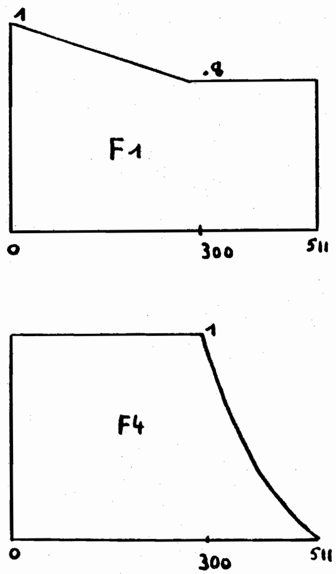
$P4 = 8s$

$P6 = 261\,Hz$ (C4)



$$P(x) = x + .71x^2 - 1.64x^3 - 6.17x^4 + 7.77x^5 + 19.3x^6 - 14.19x^7 - 24.84x^8 + 8.87x^9 + 11.08x^{10}$$





Data as in Fig 9

Fig 10: Percussive sound with complex resonance

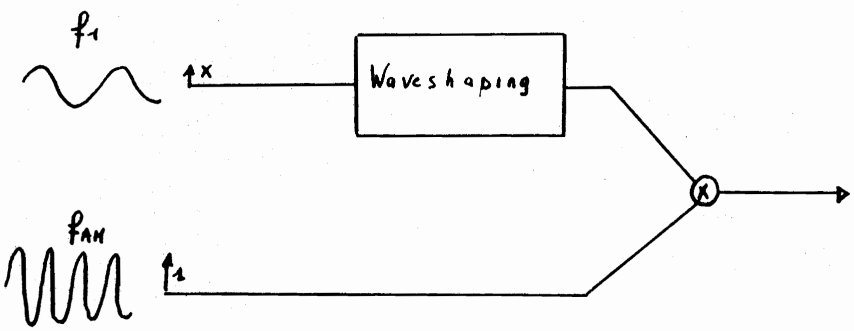
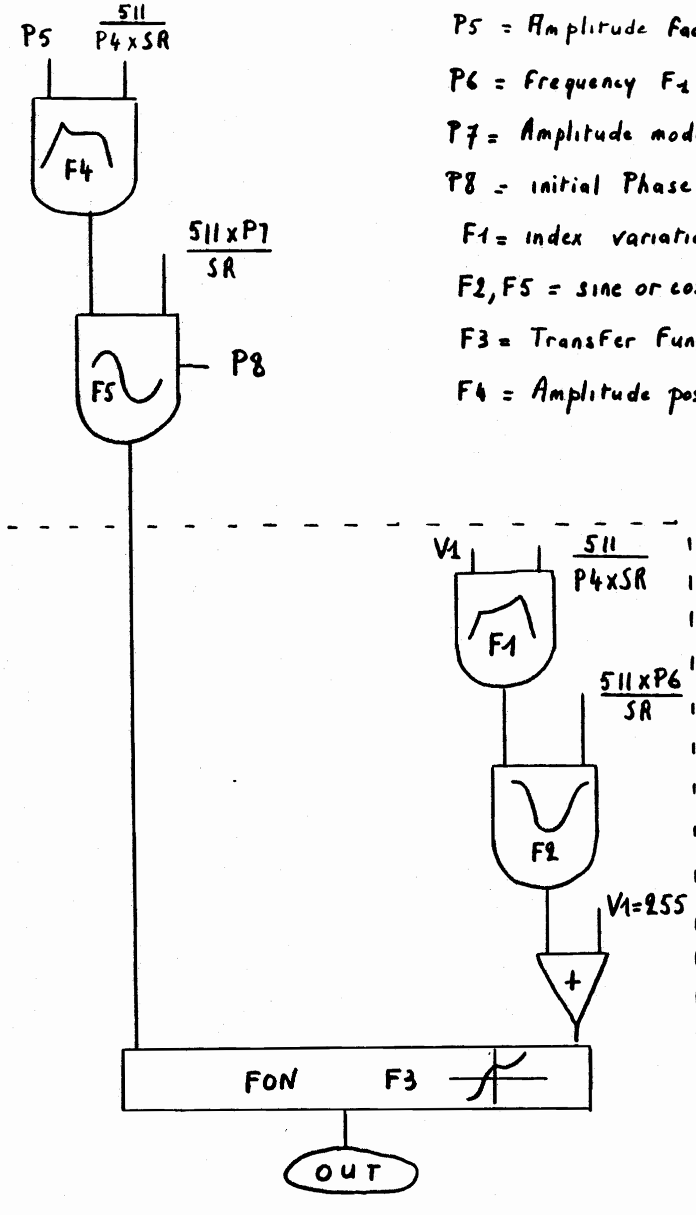


Fig 11: Amplitude modulation



P4 = Duration

P5 = Amplitude factor

P6 = Frequency F_1

P7 = Amplitude modulation frequency

P8 = initial Phase

F1 = index variation

F2, F5 = sine or cosine

F3 = Transfer Function

F4 = Amplitude post compensation

Fig 12: a MUSIC V instrument for AM

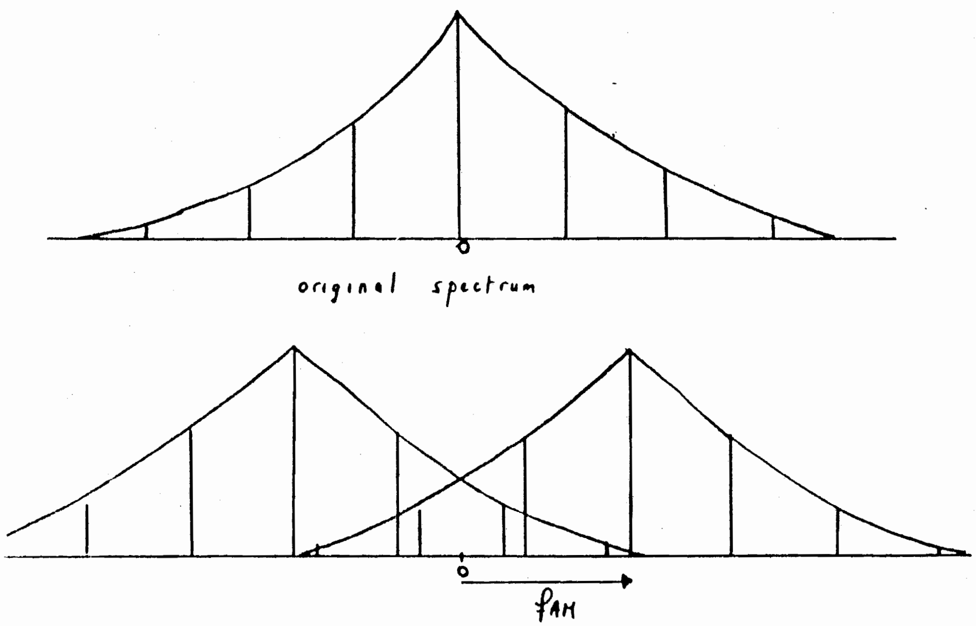


Fig 13: shifting and folding the spectrum

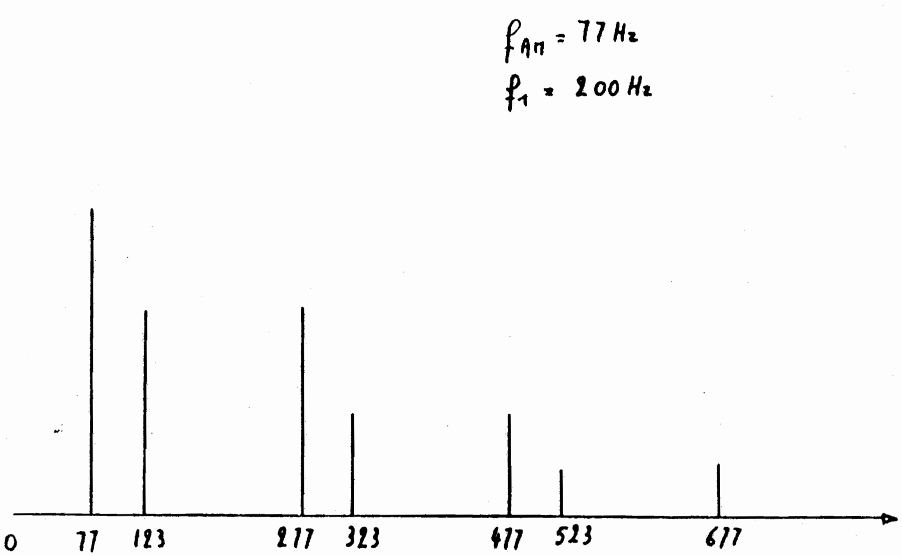


Fig 15: Inharmonic spectrum

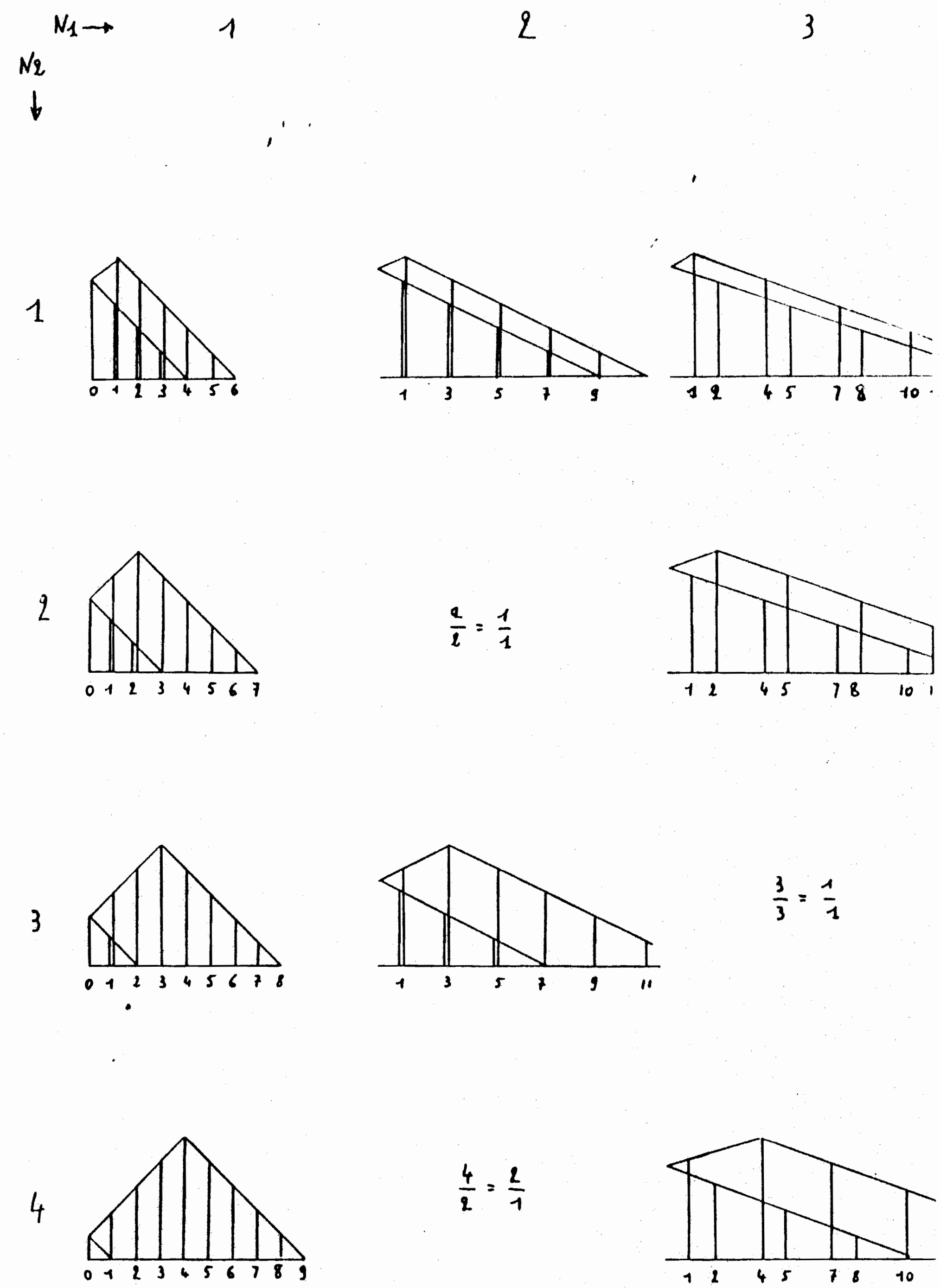
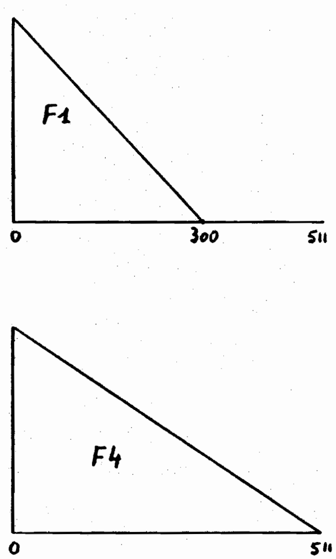


Fig 14: Aspects of spectra with $F_{am} = N_2 F_0$ and $F_1 = N_1 F_0$



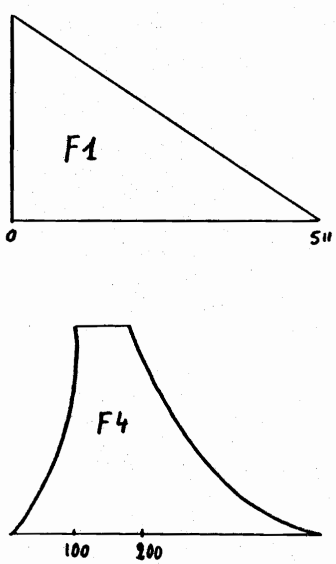
$P6 = 80\,Hz$

$P7 = 800\,Hz$

$P4 = 4\,s$

$$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6$$

Fig 16: a sound with a resonance on the 10th harmonic



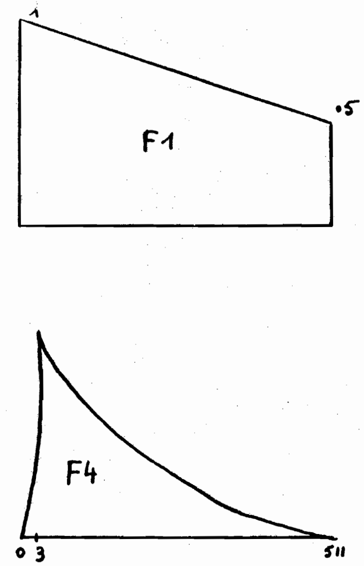
$P6 = 45\,Hz$

$P7 = 900\,Hz$

$P4 = 5\,s$

$$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$$

Fig 17: a percussive sound



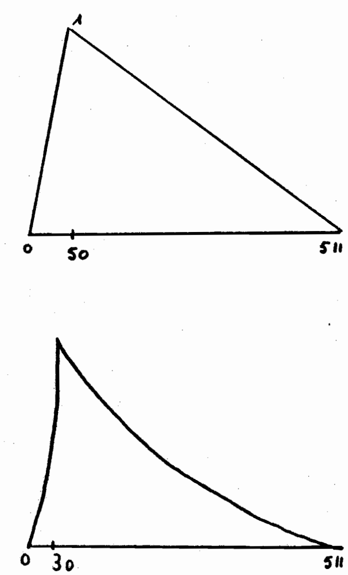
$P6 = 261\,Hz$

$P7 = 261 * 3\,Hz$

$P4 = 2\,s$

$$P(x) = 1 + x - .25x^2 - .25x^3 + .05x^4 + .05x^5$$

Fig 18: a plucked sound



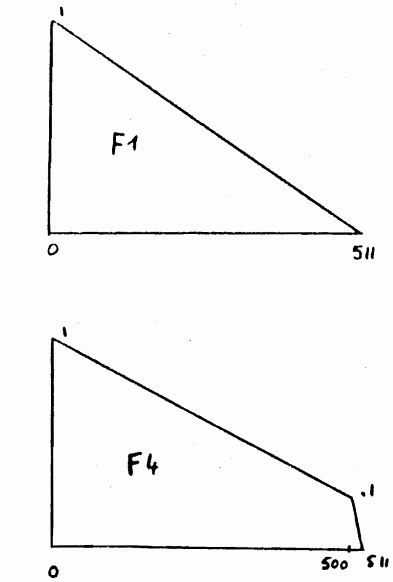
$P4 = 3\,s$

$P6 = 600\,Hz$

$P7 = 300\,Hz$

$$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$$

Fig 19: a sound with odd harmonics



P4 = 5s

P6 = 282.8 Hz

P7 = 200Hz

$$P(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$$

Fig 20 : A bell imitation

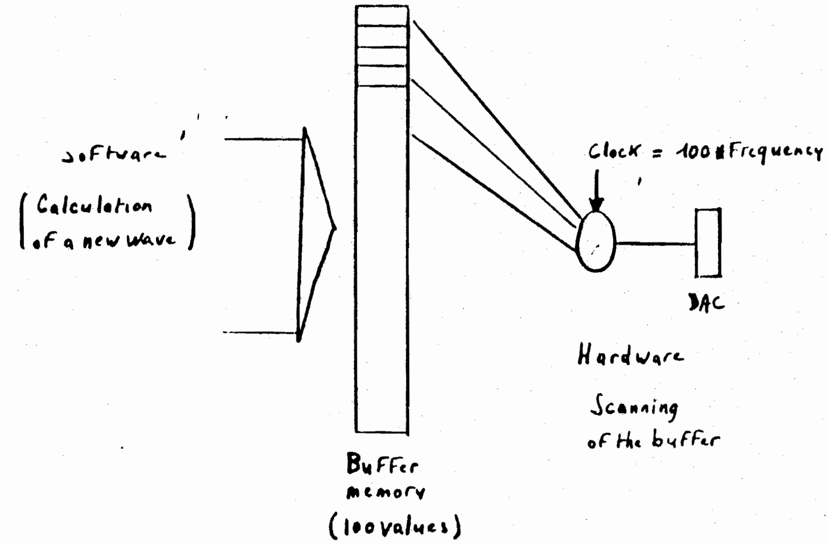


Fig 22 : A real time implementation

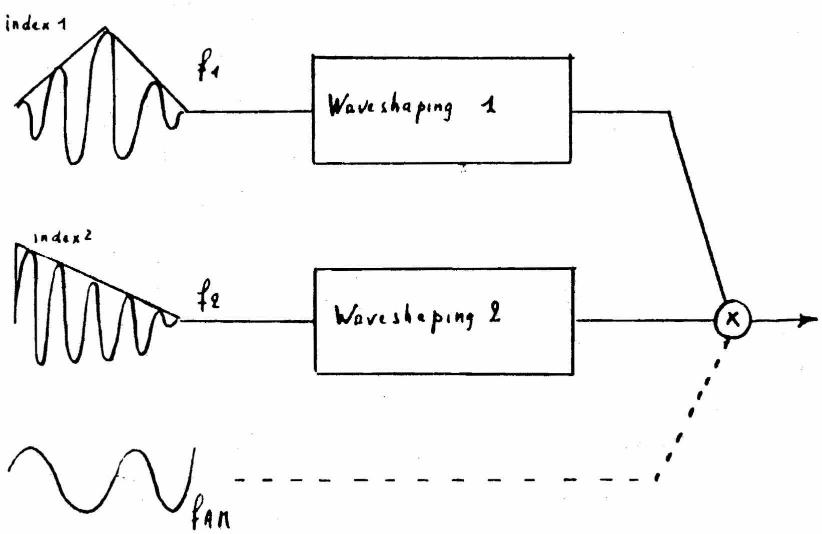


Fig 21 : Double modulation

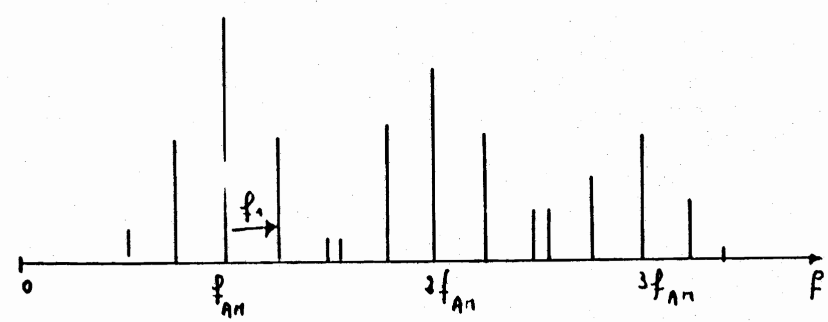


Fig 23 Modulation with a complex wave