"Cooking a Canon with RUBATO[®]" Performance Aspects of J.S. Bach's "Kunst der Fuge"

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ABSTRACT

We present and discuss an experimental investigation of Johann Sebastian Bach's "Kunst der Fuge" on the RUBATO analysis and performance platform. Three results are presented: (1) Sonification of analysis yields an immediate evidence of well-organized metrical structure. (2) Valid performances of major works of European Music can be construed in a puzzle paradigm of combinatorial construction rather than in fuzzy metaphors of creativity and inspiration. (3) Prejudice-free, analysis-driven strategies seem to yield more interesting performances than strategies which try to reach a preset expressive style-target.

1 Introduction

This paper deals with an experimental investigation of Johann Sebastian Bach's "Kunst der Fuge" on the RUBATO[®] analysis and performance platform described in [10], discussed in [9], [12], and available on the Internet from [13]. After theoretical groundwork and software implementation we feel that it is now important to elaborate the experimental paradigm in musicology, and to do this in projects of major relevance to the occidental tradition. We looked for a musical text with virtually no performance indications such that the work could be done with an utmost autonomy and exclusively dealing with the symbolic score material.

The main target of this investigation was to make sure that the overall approach which backs RUBATO is a viable framework to develop full-fledged performances. The results with "Kunst der Fuge" confirm the claim. They were established during a two-year DFG grant by one of the authors (Stange-Elbe) and can be accessed as audio and MIDI files on this ICMC's CD and from [19]. Paraphrasing Roman Jakobson's famous definition of the poetic function [5] we can state that *performance comprises projection from the axis of analysis to the axis of expression*. RUBATO operationalizes this projection model such that *valuable performance projections can be realized via intuitive puzzle-like constructions*.

2 Rational Semantics of Performance

Expressive Performance results from a complex shaping process which is applied to a given musical composition, typically defined in form of symbolic score data. Thereby, the acoustic output conveys a message which can be traced back to three main sources: music analysis, emotion, and gesture [15]. These rationales constitute the semantical background which is expressed in a given performance. Analytical semantics means that we use dynamics, articulation, agogics, and intonation to communicate analytical facts of harmonic, melodic, rhythmic, or contrapuntal nature on a rhethoric level. Emotional semantics shapes performance for a direct emotional message. Gestural semantics deals with expression of facts from physical movement (typical example: the final retard which has been modeled by mechanical kinematics, see [7]).

In human performance, all three rationales are simultaneously present, however in different degrees and depending on the precise musical situation. Accordingly, performance research has focused on all three mechanisms,

sometimes (but rarely) in an exclusive way, such as [4], where it is argued that "...we may consider emotion, motion, and music as being isomorphic." See [6] for an overview.

In our research, we have focused on the analytical perspective, however, without any claim of exclusivity or exhaustivity. Nonetheless we maintain that analysis is the most explicit semantics (in contrast to emotions), that it is a strong structural reference, and that it is a strictly musical (in contrast to gestures) and explicit instance. It therefore offers an optimal experimental playground for models of performance grammars [20].

3 Processing Bach's "Kunst der Fuge" on the RUBATO Environment

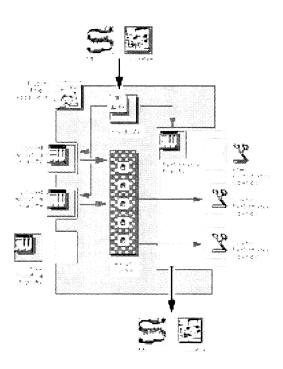


Figure 1. The RUBATO Flow chart, see text for explanation.

Our research applies the performance model as introduced in [8], [10]. For the present case of piano music, performance is described as a transformation P: SYMB -> PHYS from the four-dimensional space SYMB of symbolic events X=(E,H,L,D) with coordinates E= onset, H= pitch, L= loudness, D= duration to the corresponding four-dimensional space PHYS of physical events x=P(X)=(e,h,l,d). In this model, P is a function of so-called performance operators which implement analytical data to shape P in the sense of a "deformation of mechanical score events".

The present RUBATO platform version was realized in a SNSF grant 1992-1996 [13]. It implements this theoretical model as follows (see 1): RUBATO's in- and output files are standard MIDI or NeXT's MusicKit Scorefiles. The PrediBase system [22] of the RUBATO frame application transforms the music data into its prediacte format, including step-by-step editing. Predicates are the information units that will be processed within the RUBATO framework (the large indented rectangle on the flow chart). The process takes two directions: analysis and performance.

• For analysis, a number of analytical RUBETTES® is loaded at RUBATO's run-time. In the flow chart, these slots for analytical RUBETTES are shown to the left. A priori, their number is unlimited. To this date, three analytical RUBETTEs are available [13]. The prediacte can be piped to any such RUBETTE and yields the corresponding analysis. For future performance purposes, this analysis has to furnish a specific weight object. Intuitively, a weight associates numerical weight values with each of the prediacte's musical events as an expression of its relevance within the selected analysis. All these analytical weights are collected in a big weight system.

• To shape **performance**, the PerformanceRUBETTE has to be loaded. In our flow chart, it is inserted in the big slot to the right of the RUBATO frame. Any prediacte can then be piped into this RUBETTE and processed to a performed data set, to be saved in MIDI or Scorefile format. Shaping of a prediacte's performance makes use of the PrimavistaRUBETTE and available Performance Operators which may be added to the PerformanceRUBETTE at run-time (small slots in the PerformanceRUBETTE to the right of the flow chart). A priori, their number is unlimited. To this date, five Performance Operators are available [13].

After theoretical foundation work [8], [12] and first evidence of this approach with Schumann's "Kuriose Geschichte" [15], we were able to prove statistical significance [2], [15] for the relation between RUBATO's analytical weights and extensive performance measurements as provided by Bruno Repp's data [17], [18] for Schumann's "Träumerei". This led us to envisage a thorough investigation of Bach's "Kunst der Fuge". From the still ongoing analysis and performance work of this masterpiece of European music we shall discuss the augmentation canon (canon per augmentationem in motu contrario) No. 14. In the first edition the piece which we refer to [1], the canon is written in d minor, and consists of 104 bars in 4/4 measure, plus a 5 bar coda, totally 109 bars. The upper voice (first canon voice) starts with a variant of the main theme of this cycle, the lower voice starts on bar 5 with the first canon voice in augmentation and inversion. From bar 53, voices are exchanged. From bar 105, repetition would be possible — leading to a canon perpetuus. But the coda terminates the process. For our analysis, a literal representation of the score in a MIDI file was produced for RUBATO input, however without ornaments (trills, mordents, and turns). For performance purposes ornaments were reinserted.

4 Metrical and Motivic Analysis by Metro- and MeloRUBETTEs

The shape of analytical results produced by RUBETTES is of a very particular type. Whatever the analytical perspective, the result must be a weight, i.e. a function w_{Evt} : Evt -> \mathbf{R} defined on the set Evt of symbolic events X=(E, H, L, D) in four-space \mathbf{R}^4 , as given by the predicate which corresonds to the selected musical score. Each analytical RUBETTE yields a weight which in fact depends on Evt. We write $w_{Evt,melo}$, $w_{Evt,harmo}$ if the weight stems from melodic, metrical, or harmonic analysis. Observe that these weights are also subjected to a bunch of system parameters which intervene in the explicit description of these analyses. For example, melodic analysis needs to specify the maximal cardinality and time span of motives required to calculate the function $w_{Evt,melo}$. A thorough discussion of these parameters can be found in [8], metrical weights are discussed in [14], and melodic weights in [3], [11]. Since our canon example has only two voices, harmonic analysis has not been used in this exposition.

Intuitively, the weight value $w_{Evt}(X)$ should yield a measure of how strongly X participates in the selected analytical perspective, and relative to the total event set Evt. For events which have not been used to define the weight w_{Evt} , for example the ornamental events in Bach's canon, the discrete function w_{Evt} is interpolated by standard smooth spline methods [8] and therefore canonically applies to added events (even events that have nothing to do with the given composition!). Smoothening of weights also helps when applying infinitesimal methods in performance grammars.

The method of analytical weights requires further explanation since normally, musical analysis is rather expressed by verbal/symbolic than by numerical data. RUBATO is, however, not concerned with generic analysis but with its evaluation for the sake of performance transformations. Such transformations are defined and evaluate on real vector spaces, so there is no way out from turning analyses eventually into numerical data. Therefore it was decided to ask from any analytical RUBETTE to deliver a weight function, whatever the analytical theory. This methodology has three remarkable advantages:

- It defines a *universal interface* between analysis and performance tools.
- The investigations of performance tools can a priori rely on a unique type of analytical input, and this type fits with general tools from differential geometry and differential equations.
- Since these mathematical theories are canonical means for controlling such objects as performance transformations, the weight interface ultimately opens a wide and *coherent framework to develop performance grammars*.

Let us now turn to the concrete metrical and melodic analyses of our canon No. 14. In both analyses, a complete list of graphics can be downloaded from [19].

4.1 Metrical Weights

Metrical weights were calculated for the following event sets: $w_{Os,metro}$ stands for the upper voice Os, $w_{Us,metro}$ stands for the lower voice Us, $w_{Os+Us,metro}$ stands for the total event set Os \cup Us, and $w_{sum,metro}$ stands for the weight sum $w_{Os,metro}+w_{Us,metro}$. The system parameters are: quantization = 1/64, profile = 2, all minimal lengths down to value 2 were calculated, see [14] for technical details.

4.2 Melodic Weights

Melodic weights were calculated separately for each voice Os and Us, yielding $w_{Os,melo}$ and $w_{Us,melo}$, respectively. No melodic weight was calculated for the union Os \cup Us because of the linear construction of separate voice threads. The system parameters are: symmetry group = counterpoint, gestalt paradigm: elastic, neighborhood = 0.2. With regard to the enormous calculation amount we were limited to time span = 0.375 and motif cardinalities ranging from 2 to 7. See [3] and [11] for technical details.

We shall not discuss the analyses per se in this paper, but it should be mentioned that joint work which introduces statistical methods for the "analysis of analysis" has been published [15] is in print [2] or in preparation. We should also notice that the numerical character of weights offers a unique auditory path to grasp the contents which is wrapped in the numerical code: In fact, any weight can be viewed as a control function to drive musical parameters. For instance, the metrical weight $w_{Os,metro}$ is a function which depends only on onset, and therefore can be viewed as a function $w_{Os,metro}$: $R \rightarrow R$ on the onset axis (spline interpolation included). The value $w_{Os,metro}(E)$ can therefore be charged with a sound parameter, pitch, loudness, MIDI program change number, etc. This produces an audible image of a purely analytical fact. It is an astonishing **first result** that such a

sonification of analysis yields an immediate evidence of well-organized metrical structure,

whereas the graphical image cannot convey immediate evidence. See and hear [19] for examples.

5 Shaping Expression on the PerformanceRUBETTE

Before running into analytical performance shaping, the PrivmavistaRUBETTE can be loaded to instanciate performance indications for dynamics (e.g. crescendi), articulation (e.g. staccati), agogics (e.g. ritardandi or fermatas). For the analytical tasks, five operators which implement the analytical weights and define performance transformations are available [13]. In all cases there is a preliminary processing of weights from the existing weight system. It is called "WeightWatcher" and allows different combinations and deformations of weights. So the user may mix metrical and motivic weights, say, each with a selectable percentage of contribution to the total; the weights can be deformed in a non-linear way, the values can be flipped up-down, and the upper and lower limits of the weight ranges can be defined at will. So RUBATO's WeightWatcher is a kind of mixer where ingredients can be put together in an ample, parametrized variety. We come back to this important tool in paragraph 6 and describe now the different operators which act on a defined weight mixture.

Recall from [12], [15] that RUBATO's performance model is built upon a genealogical ramification of successively refined intermediate performances. This so-called performance stemma starts on the "prima vista" performance, the stemma's "MotherLPS" performance, and then ramifies into different sub-performances (called Local Performance Scores, LPS) of refined shape, the "daughters", "granddaughters" etc. of the prima vista performance. Each LPS is produced by a specific performance operator. Here are the different operators, for a detailed description we refer to the internet resources [13].

- The **SplitOperator** simply allows of splitting a given set of events into disjoint "grouping" subsets according to definable parameter limits in the four-space of parameters E, H, L, D.
- The **SymbolicOparator** allows to change the symbolic event parameters according to a selected WeightWatcher mixture. This really changes the score input and not the performance output. The set of affected parameters can be selected at will. The individual influence of the mixture on the selected parameters is freely adjustable.
- The **PhysicalOperator** does the same as the SymbolicOperator but on the output parameters e, h, l, d instead of E, H, L, D.
- The **TempoOperator** reshapes the already given (inherited from the mother performance) tempo curve as a function of the selected WeightWatcher mixture. This operator simultaneously influences onset and duration.

• The **ScalarOperator** is more complex. It is a kind of generalized TempoOperator which can be applied to a freely selectabel set of parameters. Its effect then produces a performance where tempi are also a function of other parameters, pitch or loudness, say. This operator and the TempoOperator imply the full-fledged theory of performance vector fields as introduced in [8], [9], [10].

Since the TempoOperator and the ScalarOperator make use of complex numerical ODE integration by Runge-Kutta-Fehlberg algorithms, applying these operators can be very time-consuming. It is, however, of reasonable speed on the Rhapsody operating system for G3 PowerMacs. An example of a stemma and its operator processes is described in [15]. The present canon No 14 was shaped by the quite simple stemma which is represented in Figure 2.

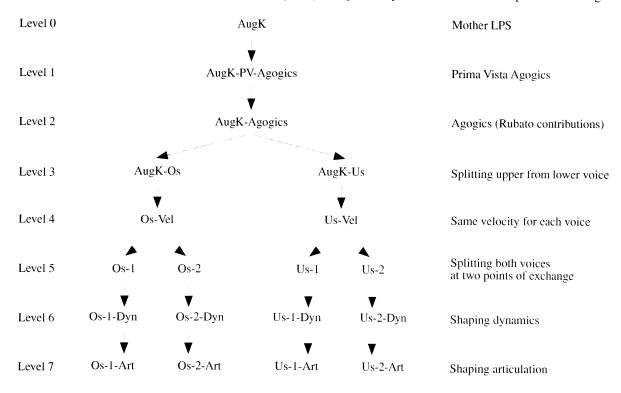


Figure 2. The stemma for the performance of Bach's canon No.14 in "Kunst der Fuge". See the text for a discussion. Here is the stemma description:

• Level 1: The prima vista agogics was shaped with the TempoOperator (calculation method: Approximate, Integrationsteps: 100) and a so-called "prima vista weight" avaliable from the PrimavistaRUBETTE. It contains the fermata on the last note (bar 109) and a little ritardando on the last half note in bar 108. This weight was used with the limits 1.0 (High Norm) and 0.2 (Low Norm).

Soundexample (bar 1-6)

- Level 2: For the shaping of rubato contributions we used a metrical weight w_{sum,metro} (minimal length of local meter = 32, see [19], Pictures Figure 1). We used the TempoOperator with the calculation method: Approximate & Adapt and Integrationsteps: 1000. Weight limits are 1.075 (High Norm) and 0.9 (Low Norm).
- Level 3: The next step was the splitting in upper and lower voice using different loudness (from MIDI velocity) values for each voice.
- Level 4: Using the SymbolicOperator (direction: Loudness) the dynamic level of each voice was set to L = 70.
- Level 5: For a different shaping both voices were separated by the SplitOperator at the points of exchange (Os: bar 55, Us: bar 52).
- Level 6: Shaping of dynamics: Together with the PhysicalOperator (direction: Loudness) four different weights were used for shaping dynamics. For the Os we used its inverted motific weight (Cardinality = 2, see [19], Pictures Figure 2a) with an influence of 60% and the limits 1.1 and 0.8 (Deformation: -0.5) for the first part and limits 1.1 and 0.7 (Deformation: 0.5) for the second part. We added its inverted metrical weight with an influence

of 40% (minimal length of local meter = 2, see [19], Pictures Figure 3a) using limits 1.15 and 0.35 (Deformation: 0.75) for both parts of Os. For Us — by analogy — we also used its inverted motific weight (Cardinality = 2, see [19], Pictures Figure 2b) with an influence of 60% and limits 0.9 and 0.6 (Deformation: 0.25) for the first part and limits 1.1 and 0.9 (Deformation: -0.5) for the second part. In the same way its inverted metrical weight (minimal length of local meter = 2, see [19], Pictures Figure 3b) was added with an influence of 40%, using limits 1.15 and 0.2 (Deformation: 1) for both parts of the lower voice.

Level 7: Shaping of articulation: Together with the PhysicalOperator (direction: Duration) only one weight was used for shaping articulation. We used the metrical weight calculated from the entire score (minimal length of local meter = 106, see [19], Pictures Figure 4) in its inverted form. For the upper voice the intensity was fixed with the limits 1.3 and 0.05 for the first part (only here with a deformation of 0.25) and limits 1.7 and 0.6 for the second part. The limits of the lower voice were 1.7 and 0.6 for the first part and 1.5 and 0.15 for the second part.

Soundexample (bar 1-6)

6 How to Project the Analytical Axis to the Expressive Axis

We are now ready to investigate the performance work on the stemma exposed in Figure 2. As a preliminary remark it is important to know that the entire stemma construction is editable in all details. Any LPS of a stemma ramification issued from a specific operator can be deleted, any weight mixture in a LPS of the stemma can be altered or replaced. This means that the subsequent ramifications will change according to the given dependences. So we may rework genealogically earlier shapings after having elaborated the stemma. This means that the PerformanceRUBETTE offers a very elastic and modular approach to shaping performance. It is never to late to come back to earlier work, the whole network is dynamic and can be accessed at any time from any LPS. Also is it allowed to reuse the same weight in any mixture for any number of LPS in the present stemma.

In the following discussion we want to describe the temporal order of shaping and then draw more general conclusions about methodology and philosophy. The construction of the canon stemma (2) can be viewed as an a priori frame to be tuned by a successive editing of the different LPS. We started on the easy level 1: prima vista agogics.

Then we turned to experiments on level 5 and 6: dynamics and articulation, however without distinguishing the voice exchange splitting points for the WeightWatcher parameters. We used motivic weights in the PhysicalOperator but recognized that these did not provide us with interesting performances for dynamics. We however kept the articulation shaping and came back to this after shaping agogics.

We then restarted on level 5: dynamics with motivic weights in the PhysicalOperator, but differentiated WeightWatcher parameters according to voice splitting, in particular non-linear deformation and up-down flipping. In a further experiement we used metrical weights in the PhysicalOperator to shape dynamics instead of motivic weights. Each experiment yielded interesting, though unbalanced performances. We next combined the two approaches and built a mix of 60% from the melodic weight approach and 40% from the metrical weight approach. This yielded a vivid, individual and very interesting dynamics, giving a good profile to structural particularities.

We then turned to level 2: agogics. We used metrical weights on the TeompoOperator and found that the sum weight $w_{\text{sum,metro}} = w_{\text{Os,metro}} + w_{\text{Us,metro}}$ yielded "logical" agogics for the entire piece.

Finally, after adding agogics on level 2, we observed that articulation had deteriorated, and we reshaped it by some minor change of WeightWatcher parameters.

This discussion deserves some comments and clarifications. *First* of all it is quite interesting that we did only use a single split for voice change. The entire local shaping was performed by the globally calculated weights. So the creation of dynamically and articulatory defined groupings was mainly caused by the analytical input and not by knowledge on bar or period groupings which were by not implemented in any of the available weights! *Second*, the stemma goes without any refinement strategies: For instance, it was not necessary to create dynamical substemmata. *Third*: The global strategy of stemma construction can be used to work "bottom-up": First do the local and then go to the global shaping.

As a whole, the shaping work seems quite easy — although a certain acquaintance has to be trained. It is comparable to a puzzle where there are many small parts, but where you have a good overview, and a good "table" to distribute the parts and to move them to find out the fitting partners. In classical performance theory [21], the

operational level of performance construction is very low. The interaction of performance aspects is not combinatorial, not reproducible and not identifiable in its details. This is not astonishing since human performance culture cannot rely on reproducible and independent management of details. Only with such a platform can you manage and study detailed contributions to the total performance.

So a **second result** of this investigation which is an elaborate contribution to experimental humanities [16] states that

valid performances of major works of European Music can be construed in a puzzle paradigm of combinatorial construction rather than in fuzzy metaphors of creativity and inspiration.

In a more down-to-earth language, the stemma approach ressembles cooking: You are given an arsenal of spices (the weights) and you must mix them to cook an interesting recipe. It's combinatorial, but it's very delicate, you need a sophisticated quantitative control (the WeightWatcher).

We have a third result that relates to the strategies in such a performance puzzle. Paraphrasing Roman Jakobson's famous characterization of poeticity (the poetic function is a projection of the paradigmatic axis to the syntagmatic axis), analytical semantics in performance can be viewed as a projection of the analytical axis to the expressive axis. This is a strong statement since it views performance as an expression of analytical insight, and it stresses the action of projecting something to obtain an output. Projection can take different directions and angles. Jakobson's characterization does not guarantee good poems, we only learn that projection is what has to happen, no further instructions are given. Likewise, good performance must rely on such a projection, and this is the essence of the above second result.

But the condition is not sufficiently specific. Form our experimental investigation we can however learn the following strategic lesson. There are *two approaches* to project weights into expression: Either we have an expressive effect in mind, from traditional knowledge or plain prejudices of how something should sound. Call this the *target-driven* strategy. In this case, a weight is only a means to achieve an interpretative effect which is already predefined. The second approach puts the analytical weights in its center, it is concerned with making a weight sound, i.e. trying to hear what the analytical insight can teach us. Call this the *source-driven* strategy. In this case, the weight is the main actor and plays no background for another driver.

In our experiments, we first applied the target-driven stragety. For example, we tried to achieve a Glenn Gould like performance and did not care about the individual weight. This one did not succeed, and we switched to the source-driven strategy which turned out to be much more successful. To restate our experience in terms of cooking: The target-driven strategy is as if you tried to cook a chicken recipe while being given a fish. The source-driven strategy is headed towards understanding and optimizing given weights and not hoped-for effects. Example A and B show the results of these strategies, when applied to bars 29-35 of the canon, see [19] (Examples for target- and source-driven performances). So the **third result** can be stated as follows:

Source-driven strategies seem to yield more interesting performance projections of the analytical axis to the expressive axis than target-driven strategies.

7 Acknowledgment

This research was parly supported by DFG habilitation grant No. STA 486/1-2.

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